Stochastic Sensitivity Analysis of Dynamic Thermal Models

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ABSTRACT

Over the past two decades many computer codes have evolved which simulate the dynamic thermal behaviour of buildings. An integral part of the validation of such simulation packages is to perform a differential sensitivity analysis where several of the input parameter values are adjusted and the induced changes in the output parameter values are used to deduce the response functions between the various input and output parameters. Unfortunately this approach suffers severe limitations and in the past such analyses have been substantially incomplete due to what has been termed the N factorial problem (basically the number of ways one can choose different sets of input parameters to adjust from a total set of N input parameters). A stochastic sensitivity analysis technique has been developed which appears to overcome this deficiency and it has been used to perform a sensitivity analysis on the UK Strathclyde thermal model ESP. Noise was added to the input parameter time series so that the auto-correlations and time delayed cross-correlations could be extracted. These have been used to evaluate the impulse response function for each parameter and hence deduce the sensitivity of each parameter of the dynamic thermal model.

Technique Stochastique d'analyse de sensibilité pour des modèles thermiques dynamiques

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ABSTRACT

De nombreux codes de calculateurs ont eté développés ce dernier temps pour la simulation de comportement dynamique thermique des bâtiments. Un contrôle de validite de tels codes doint comprendre une analyse de sensibilité différentielle. On fair varier les valeurs de quelques paramètres d'entrée, et des changements induits dans les paramètres de sortie on déduit les fonctions de réponse. Malheureusement ce procedé a des limitations sérieuses. Jusqu' ici de telles analyses ont neglige en particulier le probleme dit de N! (Soit le problème de choix d'un ensemble de paramètres à faire varier, parmi l'ensemble totale N). Un technique stochastique est presenté qui semble surmonter ce défaut. Par moyen de ce technique, une analyse de sensibilité a été effectuée pour le modèle thermique ESP (Strathclyde, GB). Par l'addition d'une composante de bruit au développement temporelle des paramètres d'entrée, on en extrait les fonctions d'auto-corrélation et de corrélation croisée retardee. La réponse impulsionelle, et par conséquent la sensibilité, à chaque paramètre du modèle thermique dynamique, ont ainsi été évaluées.

INTRODUCTION

Over the past two decades the widespread availability of increasingly powerful digital computing resources has made possible the development of large discrete multivariate computer simulation model's in most areas of scientific endeavor. Numerical solutions produced by such simulation models are becoming available for many problems that have been considered insoluable, for practical purposes, using classical theoretical methods. Confidence in the numerical solutions thus produced must remain low until the model has undergone a validation process.

Several distinct steps must be followed by the model-developer in forming the simulation model. A prototypical region of the real world must be chosen and its set of underlying physical laws and principles be identified. These physical laws may be embodied in several different mathematical formulations, from which a suitable representation of the mathematical equations is selected by the model-developer. The mathematical formulation of the underlying physical laws must then be rendered into discrete form and transformed into an algorithmic form using the axioms of a suitable computer language. The algorithms produced can finally be integrated into the simulation model on a computer which has a given precision. Many assumptions, approximations and compromises are inevitable at each of these steps. Consequently an exact replication of a reality should not be expected; rather that some confidence interval may be identified, within which the model-developer would bet that the numerical solution produced is an accurate representation of reality. Alternatively the model maybe said to be valid over some range within some specified uncertainty. This confidence interval is established during the validation phase of the simulation model's development; One of the main tools available to simulation model validators is sensitivity analysis.

Differential sensitivity anslysis is the most commonly employed method used in sensitivity analysis. Tomovic and Vukobratovic[1] define the sensitivity as the value of the partial differential of an output with respect to input. This is a local function evaluated for a particular set of input values. Further, it is a function of all the model parameters. For a complete description of the sensitivity function all orders of partial differentials need to be considered. It is usual for these partial differentials to be approximated by finite difference ratios and in order to obtain all the necessary ratios at least N! simulation runs need to be performed.

In the development of their Latin hypercube sampling[2] scheme for experimental design, McKay and co-workers used as a measure of sensitivity the partial rank correlation coefficient. This technique rank orders the input parameters with the assumption that the input parameters are related in a linear manner to the output parameters. They cannot be compared directly to the other sensitivity techniques which are local function of the parameters and will not be considered further in this work.

Recently Schruben and Cogliano[3] illustrated how the frequency response function of a multi-input multi-output simulation model could be obtained. The simulation model is driven with sinusoidal inputs at various assigned frequencies. Spectral analysis of the output signals obtained can then be

used to identify the sensitivity, in the simulation model's response, to each of the input parameters.

Stochastic Sensitivity Analysis

The present work uses [4] stochastic inputs to obtain directly the partial impulse responses of the multi-input multi-output simulation mode. In the following the input time series to the simulation model is presumed to have a deterministic part to which is added a stochastic part which is presumed to be ergodic with a zero mean. Now the output of the physical system $y(t)$, at time t, may be described in terms of the input series $\{x(t)\}$, up to time t, with a weighting sequence $\{h(\tau)\}$ which are functions of time delay. These weighting co-efficients $h(\tau)$ are usually known as the finite impulse response of the system. If the input series $\{x(t)\}$ is an arbitrary piecewise continuous function then the output of the system, $y(t)$, is related to the input of the system $x(t)$ by the convolution integral

$$y(t) = \int_{-\infty}^{\infty} h(\tau)\, x(t-\tau)d\tau = h(\tau) \otimes x(t-\tau) \qquad (1)$$

For real physical systems it is only possible for the system to respond to past inputs.

There exists a relationship between the auto-correlation $E[(x(t) - \bar{x})(x(t-\tau) - \bar{x})]$ and the cross-correlation $E[(x(t) - \bar{x})(y(t-\tau) - \bar{y})]$ in terms of the impulse response function $h(\tau)$ known as the Wiener-Hopf equation.

$$E[(x(t) - \bar{x})(y(t-\tau) - \bar{y})] = h(\tau) \otimes E[(x(t) - \bar{x})(x(t-\tau) - \bar{x})] \qquad (2)$$

An interesting feature of the Wiener-Hopf equation is that if the input auto-correlation function is a delta function the equation simplifies to

$$E[(x(t) - \bar{x})(y(t-\tau) - \bar{y})] = h(\tau) \cdot E[(x(t) - \bar{x})^2] \qquad (3)$$

This makes the evaluation of the partial impulse response function between the input x and output y simply a case of obtaining the time delayed cross-correlations between the input and output pairs of the system.

Now $x(t)$ is a random or stochastic process so that the differential must also be developed in a probabilistic framework. Calculus is based on the idea of convergence and we shall assume that the idea of mean square convergence is valid. So now it is only meaningful to consider the differential of the expectation value $E[x(t)]$ over some small range $0<t<T$. This may be considered as a time window which moves along the time series.

It can be shown[4].

$$\frac{\partial y}{\partial x} = \frac{\partial E[y(t)]}{\partial E[x(t)]} = \sum_{\tau=o}^{T} h(\tau) \qquad (4)$$

ie. the area under the curve this links Tomovic and Vukobratovic's definition of the sensitivity function at t, to the expectation value of the impulse response function up to time t. The impulse response function is related to the frequency response function by the Fourier transform. So there is an obvious correspondence with the frequency sensitivity analysis technique proposed by Schruben and Cogliano.

Results

In order to perform a complete differential sensitivity analysis for one point of the input parameter space at least N! simulations need to be performed. For many simulation models $N \sim 100$, ie. $N! > 10^{100}$. This means that differential sensitivity analysis is generally impractical and erronious conclusions may be drawn from an incomplete picture.
By contrast, using the stochastic technique to obtain the first order estimate (or any order up to N, dependent only on the storage available) only several thousand simulations need to be performed. When performing a stochastic sensitivity analysis it may not be possible or appropriate to draw the values of the input parameters of a simulation model from probability distributions. It may be that only sub-sets of the input parameters can be used. For the stochastic sensitivity analysis technique to be valid it must be demonstrated that the partial impulse response functions obtained between particular input-output pairs (or more generally input-output sets) are independent of other input parameters which are simultaneously perturbed. Here, perturbed is used in the sense that the input parameter is given a different value at each time step, this value being drawn from some probability distribution which may be time dependent.

The dynamic thermal model ESP (Strathclyde) typically uses several hundred input parameters together with stochastic boundary conditions (meteorological data). In order to extract the partial impulse response functions, suitable software has been incorporated into ESP and used in a series simulation sets performed. The input data set chosen as the test case was a two zone bungalow. A three day period was simulated using 4 time steps per hour. This three day simulation was repeated until 110 simulations had been performed. During each simulation the values of one or many input parameters was perturbed at each step as described above. One parameter was perturbed for 110 repeat simulations and the partial impulse response functions deduced. In a similar manner the same partial impulse response functions were extracted when two, ten, and four hundred input parameters were simultaneously perturbed. Figures 1a, 1b, 1c and 1d which shows that the partial impulse response functions are independent of which other inputs are perturbed. Having obtained the individual partial impulse response functions the principle of linear superposition may be tested. In order to do this the impulse response functions obtained were each driven with meteorological data and their outputs added together to produce an effective internal air temperature and surface temperature as shown in Figures 2a and 2b.

We are now able to evaluate the sensitivity of the output parameters with respect to the input parameters. For the non-zero mean case the partial impulse response functions may be evaluated using the central moment estimates for the auto-correlations and time-delayed cross-correlations.

Examples of the sensitivities, $h_{ij}(\tau)$, determined using the stochastic technique, with 3000 repeat simulations, are given in Table 1. For comparison sensitivities obtained using the differential technique are included.

In the differential sensitivity analysis only single parameters were varied and the parameter was adjusted by an amount equal to the standard deviation obtained for the same parameter in stochastic sensitivity analysis. In performing the differential sensitivity analysis it was observed that different values for the sensitivity were obtained for a parameter, say a thermophysical property when different boundary conditions were used. For this reason the same boundary conditions were used in both the differential and stochastic sensitivity analysis.

In parallel to this work analytical tests are being developed using the stochastic sensitivity technique and comparisons with multiple differential sensitivity analysis made.

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References

1. R. Tomovic and M. Vukobratovic, "General Sensitivity Theory," American Elsevier 1972.

2. M. D. McKay, W. J. Conover and D. E. Whiteman, "Report on the Application of Statistical Techniques to the Analysis of Computer Codes," Los Alamos Scientific Laboratory, LA-NUREG-6256-ms, New Mexico, 1976.

3. L. W. Schruben and V. J. Cogliano, "Simulation Sensitivity Analysis: A frequency Domain Approach", 1981 Winter Simulation Conference Proceedings, Edited by Oven, Del Josse and Shub.

4. A. D. Irving and J. P. Allen, "Stochastic Sensitivity Analysis," Submitted to Applied Mathematical Modelling.

TABLE I

Input X	Output Y	Internal Air Temperature		Internal Wall Surface Temperature	
		Differential Sensitivity	Stochastic Sensitivity	Differential Sensitivity	Stochastic Sensitivity
Dry Bulb Temperature	°C	0.418	0.522 ±0.008	0.415	0.419 ±0.006
Diffuse Radiation	Wm^{-2}	0.0127	0.0143 ±0.0005	0.0193	0.0260 ±0.0030
Direct Normal Radiation	Wm^{-2}	0.0068	0.0050 ±0.0002	0.0105	0.0115 ±0.0003
Wind Speed	ms^{-1}	-0.092	-0.091 ±0.009	-0.217	-0.273 ±0.040
Wall Layer Conductivity	$Wm^{-1}\,°C^{-1}$	0.08	0.17 ±0.10	0.58	0.55 ±0.17
Wall Layer Thickness	m	-0.96	- 7.6 ± 3.6	-3.67	-3.2 ±2.0
Wall Layer Density	kgm^{-3}	-0.00051	-0.00041 ±0.00020	-0.00054	-0.00050 ±0.00036
Wall Layer Specific Heat	$Jkg^{-1}\,°C^{-1}$	-0.00002	+0.00048 ±0.00028	-0.00002	-0.00012 ±0.00005
Wall External Surface Absorptivity		-1.42	-1.32 ±0.19	-3.81	-3.13 ±0.12

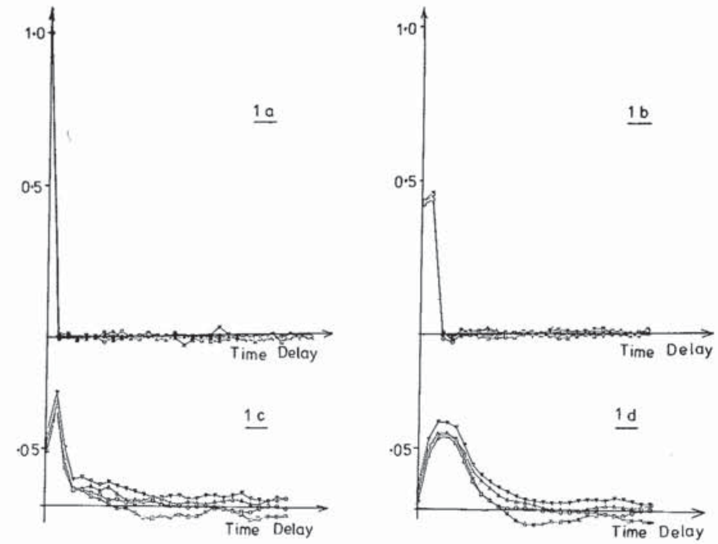

Figure: 1(a) Auto-correlation of the external dry bulb temperature; the time delayed cross-correlations between the dry bulb temperature and 1(b) the internal air temperature; 1(c) the wall internal surface temperature; 1(d) the floor surface temperature. Obtained using 110 resimulations.

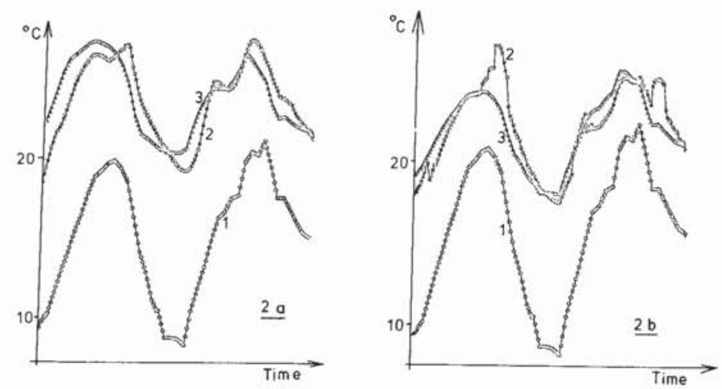

Figure: 2(a) shows the external dry bulb temperature, curve 1, and the internal air temperature as predicted by ESP, curve 2, and using the impulse response curves shown in figure 1, curve 3.

Figure: 2(b) as for figure 2(a) for the wall internal surface temperature.