# OPTIMAL CHOICE OF WASTEWATER TREATMENT TRAIN BY MULTI-OBJECTIVE OPTIMIZATION

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## ABSTRACT

A domestic wastewater treatment train consists of a series of unit operations of various types, i.e., for preliminary, primary, secondary, tertiary, and advanced treatment and residual management. Many options are available for each type of unit operations. The challenge is to select treatment trains for which the extent and reliability of treatment is high, while the capital, operation and maintenance (O&M) costs of the treatment and land area requirement is low. This proposition has been formulated as a multi-objective optimization problem. The problem was solved using a genetic algorithm to determine the pareto-optimal (‘no worse than each other) set of solutions under three conditions, i.e., when the environmental cost (E) was not constrained, and for E<75, and E<50. The results indicated that the optimal solution set contained more solutions with elaborate treatment trains when E was constrained. Furthermore, the correctness of the algorithm was demonstrated by showing that the set of optimal solutions remain approximately the same irrespective of the variations in the initial population size chosen for genetic operations.

## KEY WORDS

wastewater treatment, multi-objective optimization, environmental cost

## INTRODUCTION

Objective of wastewater treatment is to ensure that the impact of the discharge of the wastewater into the natural environment is minimized. At the same time wastewater treatment requires land and investment of funds for capital and recurring expenses. Numerous wastewater treatment options are available and more choices are becoming available everyday due to technological development. In this complex scenario, a decision maker has to choose a wastewater treatment option that is low in capital and operating cost and land requirement, while being simultaneously reliable and efficient. A methodology for choosing optimal wastewater treatment trains through formulation of a multi-objective optimization problem is presented in this paper. The input required are the capital and O&M costs and land area requirement of the unit operations of various types, and the corresponding

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measure of their operational reliability (in a 0 – 1 scale). Due to uncertainties inherent with such information, the above data may be input as fuzzy numbers. In addition, environmental cost (E) corresponding to various treatment trains is input as a normalized parameter (in a 0 – 100) scale, with E being 100 corresponding to the 'no treatment' option. In other cases, E is a function of both treatment train efficiency and reliability. The problem was solved to determine the pareto-optimal ('no worse than each other) set of solutions under three conditions, when the environmental cost was not constrained, and for E<75, and E<50.

## PROBLEM FORMULATION

As shown in Figure 1, the wastewater treatment process train consists of a set of unit operations. for some or all of the following, a). preliminary treatment (I), b). primary treatment (P), c). secondary treatment (S), d). tertiary treatment (T), e) other advanced treatments (A) and f). residuals management (R). Based on a general understanding of wastewater treatment operations, it is suggested that 11 options for treatment trains are possible, 1) no treatment, I, IP, IPS, IPST, IPSTA, IR, IPR, IPSR, IPSTR, and IPSTAR.

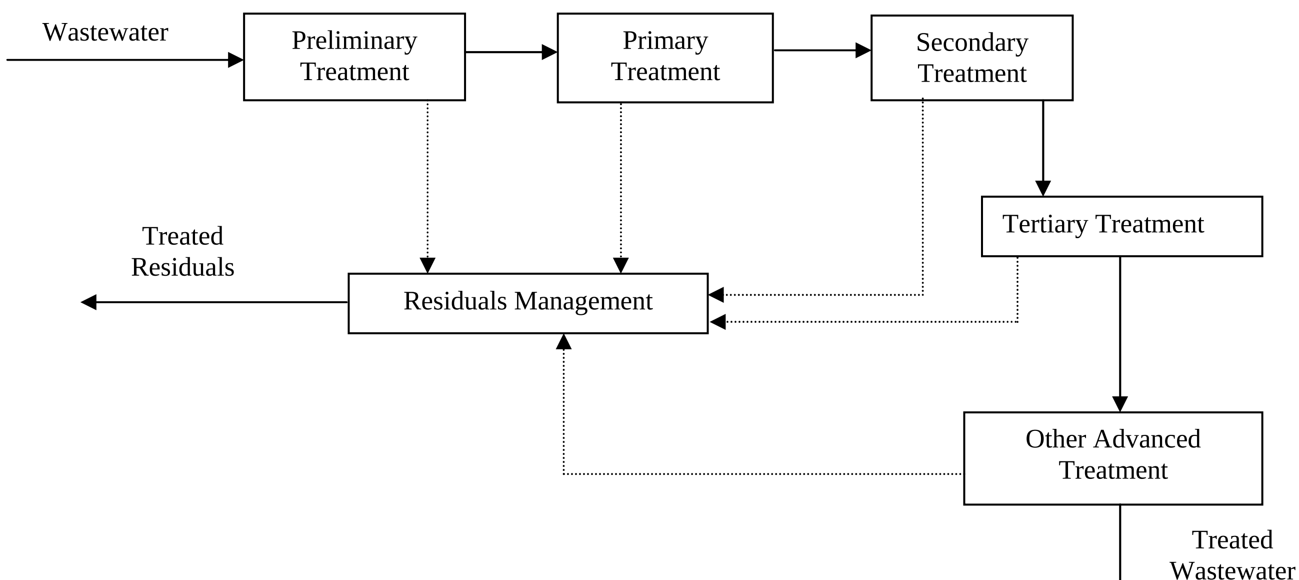


Figure 1 Conceptual Diagram of a Wastewater Treatment Train

Let each unit operation, i.e., I, P, S, R, T, A and R have g, p, s, t, a and r alternatives respectively. Thus total number of possible treatment options is N, where, $N = \sum_{i=1}^{11} N_i$, and, $N_1 = 1$, $N_2 = g$, $N_3 = g.p$, $N_4 = g.p.s$, $N_5 = g.p.s.t$, $N_6 = g.p.s.t.a$, $N_7 = g.r$, $N_8 = g.p.r$, $N_9 = g.p.s.r$, $N_{10} = g.p.s.t.r$, and $N_{11} = g.p.s.t.a.r$. Each of these 'N' alternative options for treatment are distinct in terms of capital cost ($C_C$), operation and maintenance cost ($C_O$), land area requirement (L), reliability (B) and overall environmental impact (E).

For specifying the treatment trains a set of binary variables $Y_j$ (j = 1 to 11) is defined, such that,

$$\sum_{j=1}^{11} Y_j = 1. \quad (1)$$

Then, capital cost,

$$\begin{aligned}\tilde{C}_C = \ & (\tilde{C}_c)_I.(1-Y_1) + \\ & (\tilde{C}_c)_P.(1-Y_1).(1-Y_2).(1-Y_7) + \\ & (\tilde{C}_c)_S.(1-Y_1).(1-Y_2).(1-Y_3).(1-Y_7).(1-Y_8) + \\ & (\tilde{C}_c)_T.(1-Y_1).(1-Y_2).(1-Y_3).(1-Y_4).(1-Y_7).(1-Y_8).(1-Y_9) \\ & (\tilde{C}_c)_A.(1-Y_1).(1-Y_2).(1-Y_3).(1-Y_4).(1-Y_5).(1-Y_7).(1-Y_8).(1-Y_9).(1-Y_{10}) + \\ & (\tilde{C}_c)_R.(1-Y_1).(1-Y_2).(1-Y_3).(1-Y_4).(1-Y_5).(1-Y_6)\end{aligned}$$

operation and maintenance cost, $\tilde{C}_o$ , and land area requirement, $\tilde{L}$ can also be defined similarly. we further define six other binary variables, $X_{Il}$, $X_{Pl}$, $X_{Sl}$, $X_{Tl}$, $X_{Al}$ and $X_{Rl}$, subject to the constraints below,

$$\sum_{l=1}^{g} X_{Ig} = (1-Y_1); \quad (2)$$

$$\sum_{l=1}^{p} X_{Pl} = (1-Y_1).(1-Y_2).(1-Y_7) \quad (3)$$

$$\sum_{l=1}^{s} X_{Sl} = (1-Y_1).(1-Y_2).(1-Y_3).(1-Y_7).(1-Y_8) \quad (4)$$

$$\sum_{l=1}^{t} X_{Tl} = (1-Y_1).(1-Y_2).(1-Y_3).(1-Y_4).(1-Y_7).(1-Y_8).(1-Y_9) \quad (5)$$

$$\sum_{l=1}^{a} X_{Al} = (1-Y_1).(1-Y_2).(1-Y_3).(1-Y_4).(1-Y_5).(1-Y_7).(1-Y_8).(1-Y_9).(1-Y_{10}) \quad (6)$$

$$\sum_{l=1}^{r} X_{Rl} = (1-Y_1).(1-Y_2).(1-Y_3).(1-Y_4).(1-Y_5).(1-Y_6) \quad (7)$$

Then, capital cost for each unit operations can be further resolved and written as,

$$(\tilde{C}_C)_I = X_{I1}.(\tilde{C}_C)_{I1} + X_{I2}.(\tilde{C}_C)_{I2} + \ldots\ldots + X_{Ig}.(\tilde{C}_C)_{Ig}$$

$$(\tilde{C}_C)_P = X_{P1}.(\tilde{C}_C)_{P1} + X_{P2}.(\tilde{C}_C)_{P2} + \ldots\ldots + X_{Pp}.(\tilde{C}_C)_{Pp}$$

$$(\tilde{C}_C)_S = X_{S1}.(\tilde{C}_C)_{S1} + X_{S2}.(\tilde{C}_C)_{S2} + ...... + X_{Ss}.(\tilde{C}_C)_{Ss}$$

$$(\tilde{C}_C)_T = X_{T1}.(\tilde{C}_C)_{T1} + X_{T2}.(\tilde{C}_C)_{T2} + ...... + X_{Tt}.(\tilde{C}_C)_{Tt}$$

$$(\tilde{C}_C)_A = X_{A1}.(\tilde{C}_C)_{A1} + X_{A2}.(\tilde{C}_C)_{A2} + ...... + X_{Aa}.(\tilde{C}_C)_{Aa}$$

$$(\tilde{C}_C)_R = X_{R1}.(\tilde{C}_C)_{R1} + X_{R2}.(\tilde{C}_C)_{R2} + ...... + X_{Rr}.(\tilde{C}_C)_{Rr}$$

Operation and maintenance cost, and land area requirement for various unit operations can be resolved and written similarly.

Reliability for a particular unit operation is defined as B = 1 – P(f), where P(f) is the probability of failure, representing the fractional period of time for which the unit operation is not working as per design specifications. Considering the reliability of each unit operation to be independent of the others in the treatment train, reliability of a treatment train can be represented by the product of the reliabilities of the critical unit operations, functioning of which are necessary for satisfactory functioning of the treatment train. Reliability of the treatment train is thus represented as,

$$\begin{aligned}\tilde{B} = &\left[\left(\tilde{B}_I\right)^{(1-Y_1).(1-Y_3).(1-Y_4).(1-Y_5).(1-Y_6).(1-Y_8).(1-Y_9).(1Y_{10}).(1-Y_{11})}\right] \\ &\left[\left(\tilde{B}_P\right)^{(1-Y_1).(1-Y_2).(1-Y_4).(1-Y_5).(1-Y_6).(1-Y_7).(1-Y_9).(1Y_{10}).(1-Y_{11})}\right]. \\ &\left[\left(\tilde{B}_S\right)^{(1-Y_1).(1-Y_2).(1-Y_3).(1-Y_7).(1-Y_8)}\right]. \\ &\left[\left(\tilde{B}_T\right)^{(1-Y_1).(1-Y_2).(1-Y_3).(1-Y_4).(1-Y_7).(1-Y_8).(1-Y_9)}\right]. \\ &\left[\left(\tilde{B}_A\right)^{(1-Y_1).(1-Y_2).(1-Y_3).(1-Y_4).(1-Y_5).(1-Y_7).(1-Y_8).(1-Y_9).(1-Y_{10})}\right]. \\ &\left[\left(\tilde{B}_R\right)^{(1-Y_1).(1-Y_2).(1-Y_3).(1-Y_4).(1-Y_5).(1-Y_6)}\right]\end{aligned}$$

Using the six binary variables, $X_{Il}$, $X_{Pl}$, $X_{Sl}$, $X_{Tl}$, $X_{Al}$ and $X_{Rl}$ defined earlier, reliability of a particular unit operation may be represented by,

$$\tilde{B}_I = (\tilde{B}_{I1})^{X_{I1}}.(\tilde{B}_{I2})^{X_{I2}}...........(\tilde{B}_{Ii})^{X_{Ig}}$$

$$\tilde{B}_P = (\tilde{B}_{P1})^{X_{P1}}.(\tilde{B}_{P2})^{X_{P2}}...........(\tilde{B}_{Pp})^{X_{Pp}}$$

$$\tilde{B}_S = (\tilde{B}_{S1})^{X_{S1}}.(\tilde{B}_{S2})^{X_{S2}}...........(\tilde{B}_{Ss})^{X_{Ss}}$$

$$\tilde{B}_T = (\tilde{B}_{T1})^{X_{T1}} . (\tilde{B}_{T2})^{X_{T2}} \ldots\ldots\ldots (\tilde{B}_{Tt})^{X_{Tt}}$$

$$\tilde{B}_A = (\tilde{B}_{A1})^{X_{A1}} . (\tilde{B}_{A2})^{X_{A2}} \ldots\ldots\ldots (\tilde{B}_{Aa})^{X_{Aa}}$$

$$\tilde{B}_R = (\tilde{B}_{R1})^{X_{R1}} . (\tilde{B}_{R2})^{X_{R2}} \ldots\ldots\ldots (\tilde{B}_{Rr})^{X_{Rr}}$$

Overall Environmental Impact (E) may be defined as the overall adverse impact of releasing the wastewater to the environment, either after complete, partial or no treatment. E shall be less when release is after complete or nearly complete treatment, while release without treatment or with minimal treatment will result in high E. Value of E is also dependent on the reliabilities of the individual unit operations in the selected treatment train, as all the unit operations in any treatment train will not function properly all the time. Overall environmental impact (E) is represented as, $E = \sum_{j=1,11}^{i=j} E_i . Y_j$. Depending on the set of values assigned to $Y_j$, environmental impact ($E_i$) value corresponding to only one of the 11 possible treatment trains will be considered. In order to quantify $E_i$, we first define $*E_i$ as a measure of the adverse impacts of a treatment train, working as per design specifications, on the environment. Eleven values of $*E_i$ ($*E_1$ to $*E_{11}$), corresponding to each type of treatment train may be defined. Environmental impact ($E_i$) of the treatment train is thus a function of $*E_i$ and its overall reliability (B), since the treatment train is not expected to work as per design specifications all the time. (see Biswas, 2005, for details).

The multi-objective optimization problem may thus be framed as follows,

1). Minimize $\tilde{C}_C$, $\tilde{C}_O$, $\tilde{L}$, $(-\tilde{B})$ and E, subject to the constraints given by Eq. 1 to 7

## PROBLEM SOLUTION

### SOLUTION METHODOLOGY

The problem formulated above was solved using a multi-objective optimization algorithm called NGSA (Srinivas and Deb, 1994). The values for g, p, s, t, a and r, i.e., the number of options for considered for preliminary, primary, secondary, tertiary and advanced treatment and residual management were taken as 2, 2, 8, 8, 8 and 8 respectively.

The fuzzy input values for capital cost, operation and maintenance cost, land requirement and reliability for various options of each unit operation used in solving the above problem are given in Tables 1 – 4 respectively. The $*E_i$ (I = 1, 11) was 100, 95, 88, 64, 60, 45, 80, 55, 35, 25 and 20 respectively.

Table 1: Fuzzy Capital Cost $(\tilde{C}_C)$ Values

| | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| | g = 1 | g = 2 | | | | | | |
| $(\tilde{C}_C)_{Ig}$ | (73,77) | (28,32) | | | | | | |
| | p = 1 | p = 2 | | | | | | |
| $(\tilde{C}_C)_{Pp}$ | (38,42) | (23,27) | | | | | | |
| | s = 1 | s = 2 | s = 3 | s = 4 | s = 5 | s = 6 | s = 7 | s = 8 |
| $(\tilde{C}_C)_{Ss}$ | (43,47) | (93,97) | (88,92) | (9,13) | (23,27) | (63,67) | (33,37) | (13,17) |
| | T = 1 | t = 2 | t = 3 | t = 4 | t = 5 | t = 6 | t = 7 | t = 8 |
| $(\tilde{C}_C)_{Tt}$ | (13,17) | (33,37) | (15,19) | (78,82) | (88,92) | (93,97) | (28,32) | (43,47) |
| | a = 1 | a = 2 | a = 3 | a = 4 | a = 5 | a = 6 | a = 7 | a = 8 |
| $(\tilde{C}_C)_{Aa}$ | (48,52) | (68,72) | (38,42) | (73,77) | (58,62) | (28,32) | (8,12) | (88,92) |
| | R = 1 | r = 2 | r = 3 | r = 4 | r = 5 | r = 6 | r = 7 | r = 8 |
| $(\tilde{C}_C)_{Rr}$ | (63,67) | (18,22) | (68,72) | (43,47) | (28,32) | (53,57) | (88,92) | (5,7) |

Table 2: Fuzzy Operation and Maintenance Cost $(\tilde{C}_0)$ values

| | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| | g = 1 | g = 2 | | | | | | |
| $(\tilde{C}_O)_{Ig}$ | (52,56) | (30,34) | | | | | | |
| | p = 1 | p = 2 | | | | | | |
| $(\tilde{C}_O)_{Pp}$ | (40, 44) | (88,92) | | | | | | |
| | s = 1 | s = 2 | s = 3 | s = 4 | s = 5 | s = 6 | s = 7 | s = 8 |
| $(\tilde{C}_O)_{Ss}$ | (58,62) | (28,32) | (8,12) | (88,92) | (43,47) | (93,97) | (35,39) | (18,22) |
| | T = 1 | t = 2 | t = 3 | t = 4 | t = 5 | t = 6 | t = 7 | t = 8 |
| $(\tilde{C}_O)_{Tt}$ | (38,42) | (23,27) | (68,72) | (48,52) | (15,19) | (55,59) | (5,7) | (73,77) |
| | a = 1 | a = 2 | a = 3 | a = 4 | a = 5 | a = 6 | a = 7 | a = 8 |
| $(\tilde{C}_O)_{Aa}$ | (23,27) | (63,67) | (33,37) | (13,17) | (88,92) | (10,14) | (74,78) | (58,62) |
| | R = 1 | r = 2 | r = 3 | r = 4 | r = 5 | r = 6 | r = 7 | r = 8 |
| $(\tilde{C}_O)_{Rr}$ | (88,92) | (53,55) | (28,32) | (43,47) | (13,17) | (33,37) | (15,19) | (62,64) |

Table 3: Fuzzy Land Area Requirement $(\tilde{L})$ Values

| | g = 1 | g = 2 | | | | | | |
|---|---|---|---|---|---|---|---|---|
| $(\tilde{L})_{Ig}$ | (280,320) | (480,520) | | | | | | |
| | p = 1 | p = 2 | | | | | | |
| $(\tilde{L})_{Pp}$ | (530,570) | (470,510) | | | | | | |
| | s = 1 | s = 2 | s = 3 | s = 4 | s = 5 | s = 6 | s = 7 | s = 8 |
| $(\tilde{L})_{Ss}$ | (480,520) | (355,395) | (490,530) | (440,480) | (620,660) | (700,740) | (355,395) | (780,820) |
| | T = 1 | t = 2 | t = 3 | t = 4 | t = 5 | t = 6 | t = 7 | t = 8 |
| $(\tilde{L})_{Tt}$ | (380,420) | (560,600) | (760, 800) | (480,520) | (280,320) | (630,.670) | (530,570) | (45,495) |
| | a = 1 | a = 2 | a = 3 | a = 4 | a = 5 | a = 6 | a = 7 | a = 8 |
| $(\tilde{L})_{Aa}$ | (630,670) | (720,740) | (380, 420) | (630,670) | (380,420) | (680,710) | (780,820) | (630,670) |
| | R = 1 | r = 2 | r = 3 | r = 4 | r = 5 | r = 6 | r = 7 | r = 8 |
| $(\tilde{L})_{Rr}$ | (430,470) | (630,670) | (455, 495) | (300,340) | (680,720) | (530,570) | (310,350) | (400,440) |

Table 4: Fuzzy Reliability $(\tilde{B})$ Values

| | g = 1 | g = 2 | | | | | | |
|---|---|---|---|---|---|---|---|---|
| $(\tilde{B})_{Ig}$ | (0.74,0.76) | (0.84,0.86) | | | | | | |
| | p = 1 | p = 2 | | | | | | |
| $(\tilde{B})_{Pp}$ | (0.82,0.84) | (0.79,0.81) | | | | | | |
| | s = 1 | s = 2 | s = 3 | s = 4 | s = 5 | s = 6 | s = 7 | s = 8 |
| $(\tilde{B})_{Ss}$ | (0.93,0.94) | (0.84,0.86) | (0.78,0.80) | (0.74,0.76) | (0.85,0.87) | (0.83,0.85) | (0.74,0.76) | (0.85,0.87) |
| | T = 1 | t = 2 | t = 3 | t = 4 | t = 5 | t = 6 | t = 7 | t = 8 |
| $(\tilde{B})_{Tt}$ | (0.90,0.92) | (0.90,0.92) | (0.77,0.79) | (0.84,0.86) | (0.88,0.90) | (0.89,0.91) | (0.93,0.95) | (0.86,0.88) |
| | a = 1 | a = 2 | a = 3 | a = 4 | a = 5 | a = 6 | a = 7 | a = 8 |
| $(\tilde{B})_{Aa}$ | (0.79,0.81) | (0.76,0.78) | (0.85,0.87) | (0.81,0.83) | (0.85,0.87) | (0.74,0.76) | (0.87,0.89) | (0.95,0.97) |
| | R = 1 | r = 2 | r = 3 | r = 4 | r = 5 | r = 6 | r = 7 | r = 8 |
| $(\tilde{B})_{Rr}$ | (0.74,0.76) | (0.88,0.90) | (0.93,0.95) | (0.76,0.78) | (0.89,0.91) | (0.91,0.92) | (0.80,0.82) | (0.78,0.80) |

The initial population of n feasible solution vectors (n) was between 10 and 200 for various solutions described here. The number of iterations was 200, and the population after 200 iterations was considered to be the optimal population in all cases.

### OPTIMAL SOLUTION WHEN E IS NOT CONSTRAINED

Examination of the set of optimal solution vectors obtained after 200 iterations (Table 5) indicate that some solution vectors with high adverse environmental impact (E) have been included in the set of optimal solution vectors due to the low values of $\tilde{C}_C$, $\tilde{C}_O$, $\tilde{L}$, and high $\tilde{B}$ associated with such solution vectors. It may be argued that some of these solution vectors are clearly infeasible, considering that wastewater treatment must result in some minimum reduction in adverse environmental impact, irrespective of other objectives.

Table 5: Solution to the Optimization Problem for Various Population Sizes (n)

| $\tilde{C}_C$ | $\tilde{C}_O$ | $\tilde{L}$ | $\tilde{B}$ | E | Treatment Train | Population Size, n | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| | | | | | | 50 | 40 | 30 | 20 | 10 |
| (0,0) | (0,0) | (0,0) | (1.00,1.00) | 100.00 | - | ■ | ■ | ■ | ■ | ■ |
| (66,74) | (70,78) | (1010,1090) | (0.82,0.84) | 89.32 | $I_2P_1$ | ■ | ■ | ■ | ■ | ■ |
| (134,146) | (135,147) | (1430,1550) | (0.93,0.94) | 67.56 | $I_1P_1S_5$ | ■ | ■ | | ■ | ■ |
| (71,81) | (132,142) | (1410,1530) | (0.64,0.67) | 65.97 | $I_2P_1R_8$ | ■ | ■ | ■ | | |
| (139,151) | (198,210) | (1230,1350) | (0.77,0.80) | 65.67 | $I_1P_2S_1$ | ■ | ■ | | | |
| (94,106) | (176,188) | (1430,1550) | (0.93,0.94) | 65.66 | $I_2P_2S_1$ | ■ | ■ | | ■ | ■ |
| (109,121) | (128,140) | (1490,1610) | (0.60,0.64) | 65.65 | $I_2P_1S_1$ | ■ | ■ | | | |
| (147,163) | (173,189) | (1810,1970) | (0.85,0.86) | 62.52 | $I_1P_1S_5T_1$ | ■ | | ■ | | |
| (172,188) | (136,152) | (1745,1905) | (0.85,0.86) | 62.39 | $I_2P_1S_2T_1$ | ■ | ■ | | ■ | ■ |
| (119,131) | (103,115) | (1540,1660) | (0.85,0.87) | 62.27 | $I_2P_1R_6$ | ■ | ■ | ■ | ■ | |
| (164,178) | (160,174) | (1765,1925) | (0.66,0.69) | 44.82 | $I_2P_1S_2R_8$ | | | ■ | | |
| (94,108) | (175,189) | (2030,2190) | (0.66,0.70) | 44.57 | $I_2P_1S_5R_8$ | | | ■ | | |
| (257,273) | (153,169) | (1695,1855) | (0.69,0.73) | 41.23 | $I_1P_1S_2R_6$ | ■ | ■ | ■ | ■ | |
| (212,228) | (131,147) | (1895,2055) | (0.76,0.79) | 41.19 | $I_2P_1S_2R_6$ | ■ | ■ | ■ | | |
| (142,158) | (146,162) | (2160,2320) | (0.77,0.80) | 40.94 | $I_2P_1S_5R_6$ | ■ | ■ | ■ | | ■ |
| (192,208) | (231,247) | (1760,1920) | (0.76,0.79) | 39.15 | $I_1P_2S_1R_6$ | ■ | | | ■ | |
| (162,178) | (161,177) | (2020,2180) | (0.76,0.79) | 39.08 | $I_2P_1S_1R_6$ | ■ | ■ | | ■ | |
| (107,125) | (213,231) | (2410,2610) | (0.75,0.77) | 37.89 | $I_2P_1S_5T_1R_8$ | ■ | ■ | ■ | ■ | |
| (270,290) | (191,211) | (2075,2275) | (0.69,0.73) | 34.18 | $I_1P_1S_2T_1R_6$ | ■ | ■ | ■ | | |

Shaded boxes identify the solutions that were present in the optimal set when the problem was solved for various n.

### IMPACT OF ADDING A CONSTRAINT ON E

In light of the above problem, imposition of a constraint on E such that it is below a certain maximum value in all optimal solution vectors is necessary. The solution to the problem obtained after incorporation of constraints E < 75, and E < 50 are shown in Tables 6 and 7 respectively.

Table 6: Solution to the Optimization Problem with an Additional Constraint, $E \leq 75$

| Serial No | Treatment Train | $\tilde{C}_C$ | $\tilde{C}_O$ | $\tilde{L}$ | $\tilde{B}$ | E |
|---|---|---|---|---|---|---|
| 1 | $I_2P_1S_3T_6$ | ( 247, 263 ) | ( 133, 149 ) | ( 2130, 2290 ) | ( .69, .73 ) | 66.47 |
| 2 | $I_2P_1R_8$ | ( 71, 81 ) | ( 132, 142 ) | ( 1410, 1530 ) | ( .64, .67 ) | 65.97 |
| 3 | $I_1P_2S_1$ | ( 139, 151 ) | ( 198, 210 ) | ( 1230, 1350 ) | ( .93, .94 ) | 65.67 |
| 4 | $I_2P_1S_4R_8$ | ( 80, 94 ) | ( 220, 234 ) | ( 1850, 2010 ) | ( .58, .61 ) | 47.31 |
| 5 | $I_1P_1S_2R_7$ | ( 292, 308 ) | ( 135, 151 ) | ( 1475, 1635 ) | ( .67, .71 ) | 44.28 |
| 6 | $I_2P_2S_1R_8$ | ( 99, 113 ) | ( 238, 252 ) | ( 1830, 1990 ) | ( .73, .75 ) | 42.75 |
| 7 | $I_2P_1S_4T_1R_8$ | ( 93, 111 ) | ( 258, 276 ) | ( 2230, 2430 ) | ( .52, .56 ) | 41.48 |
| 8 | $I_2P_1S_8R_5$ | ( 107, 123 ) | ( 101, 117 ) | ( 2470, 2630 ) | ( .76, .79 ) | 41.37 |
| 9 | $I_1P_2S_1R_6$ | ( 192, 208 ) | ( 231, 247 ) | ( 1760, 1920 ) | ( .85, .86 ) | 39.15 |
| 10 | $I_1P_1S_2T_1R_7$ | ( 305, 325 ) | ( 173, 193 ) | ( 1855, 2055 ) | ( .60, .65 ) | 37.69 |
| 11 | $I_2P_1S_2T_1R_5$ | ( 200, 220 ) | ( 149, 169 ) | ( 2425, 2625 ) | ( .67, .72 ) | 34.52 |
| 12 | $I_1P_1S_2T_1A_1R_7$ | ( 353, 377 ) | ( 196, 220 ) | ( 2485, 2725 ) | ( .48, .53 ) | 33.19 |
| 13 | $I_1P_2S_1T_6R_6$ | ( 285, 305 ) | ( 286, 306 ) | ( 2390, 2590 ) | ( .75, .79 ) | 31.51 |
| 14 | $I_2P_1S_8T_7A_1R_5$ | ( 183, 207 ) | ( 129, 151 ) | ( 3630, 3870 ) | ( .56, .61 ) | 29.90 |

Comparison of results shown in Table 5 with that in Tables 6 and 7 indicate that treatment trains consisting of relatively smaller number of unit operations, i.e., having lower values of $\tilde{C}_C$, $\tilde{C}_O$, $\tilde{L}$, and higher $\tilde{B}$, but having high values of E, are progressively eliminated from the set of optimal solution vectors are the constraint on E is progressively lowered, while treatment trains with relatively larger number of unit operations, which have higher values of $\tilde{C}_C$, $\tilde{C}_O$, $\tilde{L}$, and lower values of $\tilde{B}$, but have low values of E, become more numerous.

### SUMMARY AND CONCLUSIONS

The pareto-optimal sets of solutions obtained under three conditions, i.e., when the environmental cost (E) was not constrained, and for E<50, and E<75 (Tables 5, 6 and 7 respectively), indicate the following,

Table 3: Solution to the Optimization Problem with an Additional Constraint, $E \le 50$

| Serial No | Treatment Train | $\tilde{C}_C$ | $\tilde{C}_O$ | $\tilde{L}$ | $\tilde{B}$ | E |
|---|---|---|---|---|---|---|
| 1 | $I_2P_1S_4R_8$ | ( 80, 94 ) | ( 220, 234 ) | ( 1850, 2010 ) | ( .58, .61 ) | 47.31 |
| 2 | $I_2P_1S_8R_6$ | ( 132, 148 ) | ( 121, 137 ) | ( 2320, 2480 ) | ( .77, .80 ) | 40.94 |
| 3 | $I_1P_2S_1R_6$ | ( 192, 208 ) | ( 231, 247 ) | ( 1760, 1920 ) | ( .85, .86 ) | 39.15 |
| 4 | $I_2P_1S_8T_5R_8$ | ( 172, 190 ) | ( 165, 183 ) | ( 2470, 2670 ) | ( .58, .63 ) | 38.07 |
| 5 | $I_1P_1S_2T_1R_7$ | ( 305, 325 ) | ( 173, 193 ) | ( 1855, 2055 ) | ( .60, .65 ) | 37.69 |
| 6 | $I_2P_1S_4T_1R_3$ | ( 156, 176 ) | ( 224, 244 ) | ( 2285, 2485 ) | ( .62, .66 ) | 36.52 |
| 7 | $I_2P_1S_5T_1R_2$ | ( 120, 140 ) | ( 204, 222 ) | ( 2640, 2840 ) | ( .67, .72 ) | 34.53 |
| 8 | $I_1P_1S_2T_1A_1R_7$ | ( 353, 377 ) | ( 196, 220 ) | ( 2485, 2725 ) | ( .48, .53 ) | 33.19 |
| 9 | $I_2P_1S_1T_1R_2$ | ( 140, 160 ) | ( 219, 237 ) | ( 2500, 2700 ) | ( .74, .78 ) | 32.04 |
| 10 | $I_1P_2S_1T_6R_6$ | ( 285, 305 ) | ( 286, 306 ) | ( 2390, 2590 ) | ( .75, .79 ) | 31.51 |
| 11 | $I_1P_2S_1T_1R_3$ | ( 220, 240 ) | ( 264, 284 ) | ( 2065, 2265 ) | ( .78, .82 ) | 30.54 |
| 12 | $I_1P_2S_1T_1A_7R_8$ | ( 165, 187 ) | ( 372, 394 ) | ( 2790, 3030 ) | ( .57, .62 ) | 30.04 |
| 13 | $I_2P_1S_8T_7A_1R_5$ | ( 183, 207 ) | ( 129, 151 ) | ( 3630, 3870 ) | ( .56, .61 ) | 29.90 |
| 14 | $I_1P_1S_2T_5A_8R_6$ | ( 433, 457 ) | ( 226, 250 ) | ( 2605, 2845 ) | ( .64, .69 ) | 29.81 |
| 15 | $I_1P_1S_1T_5A_8R_6$ | ( 383, 407 ) | ( 256, 280 ) | ( 2730, 2970 ) | ( .71, .75 ) | 26.53 |
| 16 | $I_2P_2S_1T_1A_7R_3$ | ( 183, 207 ) | ( 316, 340 ) | ( 3045, 3285 ) | ( .68, .73 ) | 26.01 |

- When environmental cost (E) was not constrained, the optimal solution set consisted of several solutions consisting of relatively smaller number of unit operations which provide less treatment but are cheap to construct and operate.
- When E was constrained ($E \le 75$, or, $E \le 50$), treatment trains consisting of relatively smaller number of unit operations, i.e. with high values of E, were eliminated, while more elaborate treatment trains with lower E were included in the set of optimal solutions. These results suggest that to obtain low overall environmental impact values (E), treatment trains with more elaborate unit operations are necessary.

## REFERENCES

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