Teaching and Learning Based Optimization Algorithm for Optimum Design of Steel Buildings

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ABSTRACT

In this study, an algorithm is developed for the optimum design of the steel buildings. The optimum design problem is formulated according to LRFD-AISC. Design constraints include the displacement limitations, inter-story drift restrictions of multi-story frames, strength requirements for beams and beam-columns and geometric constraints. Teaching and learning based optimization (TLBO) technique, inspired by the interaction and outcome of teacher and learners, is employed to determine its optimum solution. The design algorithm developed selects optimum W sections for beams and columns of three dimensional steel frames so that aforementioned constraints are satisfied and the frame has the minimum weight. A large scale steel building has been designed by TLBO algorithm developed in order to test performance of the algorithm.

INTRODUCTION

Steel buildings are preferred in the construction of residential and commercial buildings due to their high strength and ductility particularly in regions that are prone to earthquakes. In last decades, usage of world’s resources comes into question due to growing economic crises and lack of raw materials. That’s why; the designer not only has to meet required criteria and performance of buildings, but also has to consider how to design steel buildings most economically. In other words, the designer has to consider how to optimize the design of steel buildings. However, optimum design of steel buildings is not an easy task for designers since most design problems are highly nonlinear. Moreover, they include discrete design variables and consist of complex design limitations on ultimate strength capacities of structural members, displacements, stability and geometric compatibilities. Stochastic search optimization methods are important tools for optimum design of steel building problem. These methods search optimum design by using certain strategies that are generally inspired by natural phenomena (Saka 2003 and 2007; Saka et al., 2013). TLBO is one of the recent addition to stochastic search optimization methods that is inspired by the interaction and outcome of teacher and learners, and predicated on the effect of the influence and guidance of a teacher on the output of learners (students) in a class (Rao
TLBO algorithm is a powerful technique used in engineering optimization and it has been used in engineering optimization studies (Rao et al. 2011 and 2012). TLBO algorithm is also utilized in structural optimization problems (Togan, 2012 and Makiabadi, 2013) and this study makes contribution to literature by optimizing a very big size 3-D building example.

In this study, TLBO algorithm based computer program is developed for optimum design of steel building problems. The sequence numbers of W steel section listed in steel profile table are treated as design variables. Design constraints are implemented according to the provisions of design code (LRFD, 2001) which include the displacement limitations, inter-story drift restrictions, ultimate strength requirements and geometric constraints. Four story and 428 member steel space frame building is treated as a design example problem. In order to make comparison, same building also optimized dynamic harmony search (DHS), ant colony optimization (ACO) and artificial bee colony (ABC) algorithms which were used for optimum design large scale optimization problems successfully in previous studies. (Saka et al, 2011; Aydogdu and Saka, 2012). All obtained optimum designs are compared in order to evaluate efficiency of TLBO algorithm.

MATHEMATICAL MODELING OF OPTIMUM DESIGN OF SPACE FRAME PROBLEMS

Optimum design of steel building problems are defined as the selection of steel sections for its building (frame) group members from steel section tables available in standards such that serviceability, strength and geometric limitations specified by the code of practice are satisfied. The objective function of optimum design problem is defined as the minimum weight of the steel building (frame) expressed as:

\[
\text{Minimize, } W(x) = \sum_{r=1}^{NG} m_r \cdot \sum_{s=1}^{t_r} l_s
\]

where; \(W\) is the weight of the steel building (frame), \(x\) is the vector of steel sections in the steel frame which are described as design variables, \(m_r\) is the unit weight of the steel section adopted for member group \(r\), \(t_r\) is the total number of members in group \(r\) and \(NG\) is the total number of member groups, \(l_s\) is the length of member which belongs to group \(r\). These optimization problems are subjected to design constraints functions which are described in a formula as follows:

\[
\left[ g_i(x) \right]_{i=1}^{NC} = \sum g_s(x), \sum g_d(x), \sum g_{ud}(x), \sum g_{id}(x), \sum g_{cc}(x), \sum g_{bc}(x)
\]

where; \(g_s, g_d, g_{ud}, g_{id}, g_{cc}\) and \(g_{bc}\) are the constraints functions for strength, deflection, inter-story drift, top story drift, column-to-column geometric and beam-to-column geometric constraints functions according to design code LRFD respectively. Strength constraint function is defined from inequalities given in Chapter H of LRFD-AISC as:

\[
g_s(x) = \frac{P_u}{\phi P_n} + \frac{8}{9} \left( \frac{M_{ux}}{\phi_b M_{ux}} + \frac{M_{uy}}{\phi_b M_{uy}} \right) - 1.0 \leq 0 \text{ for } \frac{P_u}{\phi P_n} \geq 0.2 \text{ and }\]

\[
g_d(x) = \frac{P_u}{2\phi P_n} + \left( \frac{M_{ux}}{\phi_b M_{ux}} + \frac{M_{uy}}{\phi_b M_{uy}} \right) - 1.0 \leq 0 \text{ for } \frac{P_u}{\phi P_n} < 0.2
\]

where, \(M_{ux}\) is the nominal flexural strength at strong axis (x axis), \(M_{uy}\) is the nominal flexural strength at weak axis (y axis), \(M_{ux}\) is the required flexural strength at strong
axis (x axis), $M_{uy}$ is the required flexural strength at weak axis (y axis), $P_n$ is the nominal axial strength (Tension or compression) and $P_{u}$ is the required axial strength (Tension or compression) for member i. Deflection constraints are calculated by using

$$g_d(x) = \frac{\delta_{jl}}{L/Ratio} - 1.0 \leq 0 \quad (j=1,2,\ldots,n_{sm}, l=1,2,\ldots,n_{lc})$$

where $\delta_{jl}$ is the maximum deflection of $j^{th}$ member under the $l^{th}$ load case, L is the length of member, $n_{sm}$ is the total number of members where deflections limitations are to be imposed, $n_{lc}$ is the number of load cases. Top story drift constraint function is given as:

$$g_{td} = \frac{\delta_{jl}^{top}}{H/Ratio} - 1.0 \leq 0 \quad (j=1,2,\ldots,n_{jtop}, l=1,2,\ldots,n_{lc})$$

Inter story drift constraint function is given as:

$$g_{id} = \frac{\delta_{jl}^{oh}}{h_{st}/Ratio} - 1.0 \leq 0 \quad (j=1,2,\ldots,n_{st}, l=1,2,\ldots,n_{lc})$$

In these equations, $H$ is the height of the frame, $n_{jtop}$ is the number of joints on the top story,$\Delta_{jl}^{top}$ is the top story displacement of the $j^{th}$ joint under $l^{th}$ load case, $n_{st}$ is the number of story, $\Delta_{jl}^{oh}$ is the story drift of the $j^{th}$ story under $l^{th}$ load case, $h_{st}$ is the story height and Ratio is the limitation ratio for lateral displacements. Range of drift limits by first-order analysis is 1/750 to 1/250 times the building height $H$ with a recommended value of $H/400$. Two types of geometric limitations, called column to column geometric constraints (CCGC) and beam to column geometric constraints (BCGC) are included in the design problem. CCGC equations are defined as:

$$g_{cc}(x) = \sum_{i=1}^{n_{ccj}} \left( \frac{m_i^a}{D_i^a} - 1.0 \right) \quad \text{and} \quad \sum_{i=1}^{n_{ccj}} \left( \frac{m_i^b}{D_i^b} - 1.0 \right) \leq 0$$

BCGC functions are described as:

$$g_{bc}(x) = \sum_{i=1}^{n_{bj1}} \left( \frac{B_f^i}{D_{ci} - 2t_{bi}} - 1.0 \right) \leq 0 \quad \text{or} \quad \sum_{i=1}^{n_{bj2}} \left( \frac{B_f^i}{B_f^i} - 1.0 \right)$$

In these equations, $n_{ccj}$ is the number of CCGC defined in the problem, $m_i^a$ is the unit weight of W section selected for above story, $m_i^b$ is the unit weight of W section selected for below story, $D_i^a$ is the depth of W section selected for above story, $D_i^b$ is the depth of W section selected for below story, $n_{bj1}$ is the number of joints where beams are connected to the web of a column, $n_{bj2}$ is the number of joints where beams connected to the flange of a column, $D_{ci}$ is the depth of W section selected for the column at joint $i$, $t_{bi}$ is the flange thickness of W section selected for the column at joint $i$, $B_f^i$ is the flange width of W section selected for the column at joint $i$ and $B_f^i$ is the flange width of W section selected for the beam at joint $i$. (See Figure 1).
Based on the teaching–learning process, a mathematical model of a novel optimization technique called Teaching–Learning-Based Optimization (TLBO) was developed by Rao et al., 2011. TLBO algorithm is inspired by the interaction and outcome of the teacher and learners and this method has been built on the effect of the influence and guidance of a teacher on the output of learners (students) in a class. The method is considered in two complementary components which are the teaching and learning phases. In the first phase, the highly learned person (learner) is considered as a teacher to share his/her knowledge and experience with the other learners. Obviously, the quality of the teacher has an important impact on the outcome of the class. Qualified teachers acting as a trainer, help the learners to get better results in terms of their grades. Here, output of learners or class can be considered in terms of their results or grades. Learning phase can be considered as sharing knowledge and interaction of learners among themselves. The optimum design algorithm using TLBO in the current work is explained with stepwise manner as follows.

Figure 2(a) shows the distribution of grades obtained by learners of two different classes evaluated by two different teachers, T1 and T2. Same merit level learners in two different classes are trained with the same content. The marks obtained by the learners are taught by teachers T1 and T2. The distribution may have a skewness in actual practice, it is assumed that it has a normal distribution and defined as

\[ f(X) = \frac{1}{\sigma \sqrt{2\pi}} e^{-\frac{(x-\mu)^2}{2\sigma^2}} \]

where \( \sigma^2 \) is the variance, \( \mu \) is the mean and \( x \) is any value for which the normal distribution function is required. It can be said that teacher T2 is better than teacher T1 in terms of teaching because curve-2 represents better results than curve-1. It is seen from Fig. 2(a) that a good teacher produces a better mean for the results of the learners when their means (M2 for curve-2 and M1 for curve-1) are compared.

During Teaching Phase, the first part of the algorithm, where learners learn through the teacher, the learners in the class are trained by the teacher to increase their mean result of the classroom from any value M_A to teacher’s level T_A (see Figure 2(b)). Actually, it is not possible in practice but a teacher can improve the mean performance of the class from M_A to any other value M_B which is better than M_A depending on teacher’s capability. In this step, M_i is considered as the mean and T_i is considered as the teacher at any iteration i. Now T_i will try to improve mean M_i towards its own level, so the new mean will be T_i designated as M_{new} and the
The difference between the old mean and new mean is used to update the solution and is given by (Rao et al., 2011);

$$x_{(i)}^0 = x_{j}^{\text{min}} + \text{Random} \times (x_{j}^{\max} - x_{j}^{\text{min}})$$

Obtained solutions are sorted by their performance (objective function value) and the learner which has the best grade or performance is selected as a teacher for the teaching phase of the first generation.

$$\text{Difference Mean}_i = r_i + (M_{\text{new}} - T_f M_i)$$

In this equation, $T_f$ is the teaching factor that decides the value of mean to be changed, and $r_i$ is the random number in the range $[0, 1]$. Value of the teaching factor is generated randomly during the algorithm in the range of 1-2 which is a heuristic step and it is decided with equal probability.

$$T_f = \text{round}[1 + \text{Random}(0,1)[2,1]]$$

Figure 2. Model for the distribution of marks obtained for a group of learners

For this value, 1 corresponds to no increase in the knowledge level and 2 corresponds to complete transfer of knowledge. The values between 1 and 2 indicate amount of transfer level of knowledge and the transfer level of knowledge changes depending on the learners’ capabilities. Based on this Difference Mean, the existing solution is modified by the following expression;

$$x_{\text{new},i} = x_{\text{old},i} + \text{Difference Mean}$$

In the second part of the algorithm, learners can also increase their knowledge by means of interaction among themselves according to teaching–learning process. So, learners are randomly interacted to learn something new with other learners in the class for enhancing his or her knowledge. Generally, a learner learns new things if the other learner has more knowledge than him or her. At any iteration $i$, sharing new information between the learner $i$ and $j$ in the class can be expressed mathematically;

For $i = 1:P_n$

- $X_i$ and randomly selected learner $X_j$, where $i \neq j$
- $X_{\text{new},i} = X_{\text{old},i} + r_i (X_i - X_j)$ if $f(X_i) < f(X_j)$
- $X_{\text{new},i} = X_{\text{old},i} + r_i (X_j - X_i)$ if $f(X_j) < f(X_i)$

EndFor

At the end of this evaluation, if $X_{\text{new}}$ gives better function value, it is accepted. The procedure for the implementation of TLBO is summarized below:

Step-1: Initialize the method parameters and define the optimization problem.

Step-2: Initialize the population and design variables of the optimization problem randomly and evaluate them.
Step-3: Teacher Phase. Assign the best learner as a teacher and calculate mean result of learners in the classroom. Evaluate the difference between current mean result and best mean result by utilizing the teaching factor \( T_i \). Update the learners’ knowledge with the help of teacher’s knowledge.

Step-4: Update the learners’ knowledge by utilizing the knowledge of some other learners. The procedure from step 2 to 4 is repeated until maximum iteration is reached.

**DESIGN EXAMPLE: FOUR-STOREY, 428-MEMBER STEEL BUILDING**

The design problem of this study is a 428-member steel space building. Plan and side views of this building are illustrated in Figure 3. Structural system of this building is treated as a space frame. The frame has 172 joints and 428 members which are collected in 20 independent member groups. The member grouping of the frame is illustrated in Table 1. The frame is subjected to gravity loads as well as lateral loads that are computed according to ASCE 7-05. The design dead and live loads are taken as 2.88kN/m\(^2\) and 2.39kN/m\(^2\) respectively. Basic wind speed is considered as 85mph (38 m/s) for the wind load. The following load combinations are considered in the design of the frame according to the code specification LRFD, 2001 and ASCE 7-05: 1.2D+1.6L+0.5S, 1.2D+0.5L+1.6S, 1.2D+1.6WX+L+0.5S and 1.2D+1.6WX+L+0.5S where D is the dead load, L represents the live load, S is the snow load and WX, WZ are the wind loads in the global X and Z axis respectively. The drift ratio limits for this example are taken as 0.875 cm for inter story drift and 3.5 cm for top story drift. Maximum deflection of beam members is restricted as 2.0 cm.

![Figure 3. Plan and side views of four-story, 428 member space frame](image)

<table>
<thead>
<tr>
<th>Story</th>
<th>Side beam</th>
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<th>Corner Column</th>
<th>Side Column</th>
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Optimum design of 428 member steel building problem is solved by presented algorithms 30 times using different seed values in each design. The following
parameters are used in optimization algorithms: Maximum generation=85, Population Size=60, teaching factor=1.0 and maximum iteration number=50,000. The average weight and standard deviation of the 30 best designs of each run are obtained as 1510.14 and 9.21 respectively. The weights of best designs having the smallest weight between 30 best designs are 1503.91 kN. Obtained results are compared with optimum designs that are obtained by DHS, ACO and ABC algorithms. W sections of the optimum design for TLBO algorithm are shown in Table 2. Maximum constraints values and minimum frame weights for each algorithm are illustrated in Table 3. It is apparent from these tables that weight of best design obtained by using the TLBO algorithm (1503.91kN) is 0.55% lighter than the design of ABC algorithm (1512.11kN), 1.45% lighter than the design of DHS algorithm (1526.01 kN) and 4.61% lighter than the design of ACO algorithm (1573.2kN). Search histories of these solutions are illustrated in Figure 4.

**Table 2. TLBO algorithm design results for 428 member steel building**

<table>
<thead>
<tr>
<th>Member Group Number / W Section</th>
<th>1</th>
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<th>3</th>
<th>4</th>
<th>5</th>
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**Table 3. Maximum constraints values and minimum frame weights**

<table>
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<tr>
<th>Algorithm</th>
<th>Minimum weight(KN)</th>
<th>Maximum top story drift (cm)</th>
<th>Maximum Inter-story drift (cm)</th>
<th>Maximum strength constraint ratio</th>
<th>Maximum number of iterations</th>
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</table>

![Figure 4. Search histories of 428 member steel building](image)
CONCLUSION

In this study, an optimum design algorithm which is based on TLBO algorithm is developed for optimum design of steel building problems. Large scale steel building (428 member steel building) is designed in order to test efficiency of TLBO algorithm for optimum design of steel building problems. Optimum design obtained from the TLBO algorithm is compared to those attained by the ABC, DHS and ACO algorithms. The inspection of the design histories and design result tables clearly indicates that the performance of TLBO algorithm is better than the ABC, DHS and ACO algorithms. Therefore it can be concluded that, TLBO is a robust and efficient approach that can be effectively used to determine the optimum designs of steel buildings.

REFERENCES