Flowshop Scheduling Model of Multiple Production Lines for Precast Production

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Abstract
Scheduling of precast production is essential for prefabricated building projects due to its direct impact on the construction progress. To generate appropriate schedules of precast production, various researches have been conducted. However, they mainly focused on the scheduling of a single production line which results in the difficulties in application to actual workshops of multiple production lines. To solve the problem, this paper proposes the Flowshop Scheduling Model of Multiple production lines for Precast production (MP-FSM for short here after). Firstly, the optimization objectives and constraints of flowshop scheduling of multiple production lines of precast production is analyzed, since it is essentially an optimization problem. Secondly, the MP-FSM is established by formulating the optimization objectives and constraints. Finally, a basic method for flowshop scheduling of precast production is established based on the MP-FSM. It is concluded that optimal schedules can be obtained by employing the method.

Keywords: precast production, flowshop scheduling, optimization

1 Introduction
Precast concrete structures have advantages in the efficiency and quality of construction because of the novel technologies employed in precast production, which is, therefore, critical for such type of structures. Normally, production planning includes master production scheduling, material requirement planning and shop floor scheduling, and is essential for precast production. Therein, detailed from material requirement plans, shop floor schedules assign the production tasks to specific workshop sections, teams or even persons. They directly impact the delivery time of precast components and, thus, play a significant role in production management. However, in most cases, they are manually prepared and left not optimized, which consequently results in production capability waste and high production cost.

To solve the problem, numerous researches have been conducted. Chan et al. (2000, 2002) proposed the flowshop sequencing model named FSSM for precast production, in which characteristics such as the parallel processing capability of curing rooms and the requirement of uninterruptible activities like casting and curing are considered. Besides the common optimization objective of makespan minimization employed in traditional flowshop problems, FSSM also targeted at minimizing the default penalty and inventory cost.

Considering the characteristics of bespoke precast production, Benjaoran et al. (2005) studied the impacts of the quantity of molds on shop floor schedules of precast production and proposed the flowshop scheduling model named BP-FSSM for bespoke precast production by specializing the FSSM on its optimization objectives and constraints. A planning system called “Artificial Intelligence Planner” was, then, developed based on the BP-FSSM, which enables semi-automatic quantity take-off, productivity estimation and shop floor scheduling of bespoke precast production (Benjaoran, et al. 2006).
The buffer size between work stations in a production line cannot be assumed as infinite during shop floor scheduling due to the large size of precast components. Meanwhile, it was ignored in the previous researches. Hence, Ko et al. (2010) modified the optimization constraints of the existing scheduling models and, accordingly, developed a genetic algorithm based decision support system for precast production planning. Some other researchers have also made inspiring achievements (Leu, et al. 2001, 2002; Zhai, et al. 2006; Kjalil, et al. 2013).

Nevertheless, limitations still exist in the field. First of all, the existing scheduling models are formulated from a single production line and cannot be pragmatically applied to actual workshops of multiple production lines, because shop floor scheduling of them is not the simple iteration of that of each production line and the shared resources, for example, molds must be allocated to each production line before scheduling. Second, the types of the precast components produced in turn are not continuous in the schedules generated by existing scheduling models and frequent changes in the types of the precast components produced in turn result in substantial equipment adjustments and operation changes so that production errors occur and the production efficiency is reduced. Finally, the quantity of production pallets is ignored as a constraint of shop floor scheduling in the existing scheduling models which may result in the infeasibility of the schedules generated based on them.

This paper proposes the Flowshop Scheduling Model of Multiple production lines for Precast production (MP-FSM for short here after) in which the constraint of the quantity of shared resources such as molds and production pallets and the type continuity of the precast components produced in turn are considered. In the following, Section 2 analyses the flowshop scheduling of multiple production lines of precast production. Based on the analysis, Section 3 formulates MP-FSM. Then, Section 4 establishes a basic method of flowshop scheduling based on MP-FSM to verify the model. Finally, Section 5 concludes the paper.

2 Analysis on flowshop scheduling of multiple production lines of precast production

Precast production can be divided into two categories according to the difference in production organization in the shop floors, namely flowshop production and fixed location production. The former divides the precast production as six processes, namely molding, placing of rebars and embedded parts, casting, curing, mold stripping and finishing. Each of the process is handled in a particular workstation by a particular team. A precast component is completed after passing all the different workstations. In the later, the division of precast production is similar, while all the processes are handed in fix workstations by same or different teams. Comparing with flowshop production, fix location production is appropriate for more types of precast components, especially special-shaped ones. But its production efficiency and resource utilization rate is relatively low. Furthermore, increasing types of precast components become capable to be produced in flowshop production lines thanks to the development of technologies. Therefore, this paper focus on the flowshop scheduling of multiple production lines of precast production.

The problem is generally described as follows. A precast workshop is equipped with $Q_{\delta}$ ($\delta$ is the type of the molds) molds of each type, $P$ production pallets and $L$ production lines with fixed production processes. There are six workstations in each production line for the aforementioned six processes of precast production respectively. The curing rooms, i.e. the curing workstations, are capable to handle $Y_{\delta}$ ($\delta$ is the number of a production line) precast components simultaneously. Totally, $N$ precast components of multiple types are produced in the workshops under some constraints. The aim is to figure out a floor shop schedule meeting some optimization objectives.

The flowshop scheduling of multiple production lines is, essentially, a multi-objective optimization problem with variables of the allocation of precast components to the production lines and the processing sequence of them in each production line. This paper analyses and summarizes the optimization objectives and constraints, as shown in Table 1, based on literature review and site investigation. Since the items No. 1, 2, 3, 5, 6, 7, 8 have been presented in detail in existing researches already (Chan, et al. 2002, Benjaoran, et al. 2005, Ko, et al. 2010), this paper discuss only the items No. 4, 9, 10 in the following.
Table 1 Optimization objectives and constraints of flowshop scheduling of multiple production lines of precast production

<table>
<thead>
<tr>
<th>No.</th>
<th>Classification</th>
<th>Items</th>
</tr>
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<tbody>
<tr>
<td>1</td>
<td>Optimization</td>
<td>Minimization of workstation idle time</td>
</tr>
<tr>
<td></td>
<td>objectives</td>
<td></td>
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<tr>
<td>2</td>
<td></td>
<td>Minimization of default penalty and inventory cost</td>
</tr>
<tr>
<td>3</td>
<td></td>
<td>Minimization of makespan</td>
</tr>
<tr>
<td>4</td>
<td>Optimization</td>
<td>Type continuity of the precast components produced in turn</td>
</tr>
<tr>
<td></td>
<td>constraints</td>
<td></td>
</tr>
<tr>
<td>5</td>
<td></td>
<td>Constraint of productivity</td>
</tr>
<tr>
<td>6</td>
<td></td>
<td>Constraint of the size of curing rooms</td>
</tr>
<tr>
<td>7</td>
<td></td>
<td>Constraint of the eight-hour day working</td>
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<tr>
<td>8</td>
<td></td>
<td>Constraint of the buffer size between workstations</td>
</tr>
<tr>
<td>9</td>
<td></td>
<td>Constraint of the quantity of molds</td>
</tr>
<tr>
<td>10</td>
<td></td>
<td>Constraint of the quantity of production pallets</td>
</tr>
</tbody>
</table>

2.1 Type continuity of the precast components produced in turn

As mentioned before, frequent changes in the types of the precast components produced in turn result in substantial equipment adjustments and operation changes which are harmful to the production efficiency and quality. Thereby, it is critical to maintain the type of the precast components produced in turn continuous, as shown by the item No. 4 in Table 1.

In most cases, the shift work system is employed in the workshops of precast production. Normally, a work team is supposed to work eight hours in a shift. So reducing the type changes of the precast components produced in each shift is the key to keeping the type continuity of the precast components produced in turn.

2.2 Constraint of the quantity of molds

Molds are essential during precast production. However, the quantity of molds is always limited, especially that of bespoke precast components due to the difficulties in investment apportionment. So it is of importance to ensure that the molds of each type are always more than the precast components which are of the same type and being produced in each production line respectively, as shown by the item No. 9 in Table 1. Otherwise, the schedules turn out to be infeasible.

Since molds are shared by all the production lines, the rations of the molds in each of them are still not determined before flowshop scheduling. Thereby, molds allocation is of the essence. Yet, the demand of molds varies from time to time in both the type and the quantity during precast production especially for the mixed production (Leu, et al. 2002). To make full use of the molds, the rations of the molds of each type should also be time-dependent.

2.3 Constraint of the quantity production pallets

Production pallets are the carriers of precast components during the production processes. So, it is significant to ensure that the production pallets are always more than the precast components which are being produced in each production line respectively, as shown by the item No. 10 in Table 1. Otherwise, the schedules turn out to be infeasible.

The allocation of production pallets is important as they are also a kind of shared resources. Meanwhile, since production pallets work with molds in pairs, the allocation plans of production pallets should remain coordinated with those of molds to avoid resource waste.

3 Flowshop scheduling model of multiple production lines for precast production

This paper establishes the flowshop scheduling model of multiple production lines for precast production named MP-FSM in which both the optimization objectives and the optimization constraints shown in Table 1 are formulated.
3.1 Optimization objectives

The optimization objectives in MP-FSM contain minimization of Workstation Idle time (WI for short here after), minimization of Default penalty and Inventory cost (DI for short here after), minimization of Makespan (MS for short here after) and Type Continuity of the precast components produced in turn (TC for short here after) corresponding to the optimization objectives listed in Table 1.

Since the first three optimization objectives of the scheduling models for a single production line have been presented in the existing researches already (Chan, et al. 2002, Benjaoran, et al. 2005, Ko, et al. 2010) and those for MP-FSM show little change, they are only briefly presented in this paper.

The workstation idle time, the default penalty and inventory cost increase linearly with the production lines. Hence, these indexes of multiple production lines are obtained by summarizing the corresponding indexes of each production line, the calculation methods of which are already given in existing researches (Benjaoran, et al. 2005). The first two optimization objectives are, accordingly, expressed as Equations (1) and (2). The makespan of multiple production lines is the maximum of that of each production line, the calculation method of which is already given in existing researches (Chan, et al. 2002). The third optimization objective is, accordingly, expressed as Equation (3).

\[
\begin{align*}
\text{Min } f_{WI} & = \sum_{i=1}^{n} \sum_{k=1}^{6} \left[ C\left(J_{i,n_i}, M_{i,k}\right) - \sum_{i=1}^{n_i} P_{i,k}\right] \\
\text{Min } f_{DI} & = \sum_{i=1}^{n} \left( \sum_{j=1}^{n_i} \tau_{i,j} \ast \text{Max}\left[0, C\left(J_{i,j}, M_{i,6}\right) - d_{i,j}\right] + \sum_{i=1}^{n_i} \varepsilon_{i,j} \ast \text{Max}\left[0, d_{i,j} - C\left(J_{i,j}, M_{i,6}\right)\right]\right) \\
\text{Min } f_{MS} & = \text{Max}_{i=1}^{n} \left[C\left(J_{i,n_i}, M_{i,6}\right)\right]
\end{align*}
\]

Where: \(N^+\) stands for all the positive integers;
\(n_i\) is the quantity of precast components produced in production line \(i\) (\(1 \leq i \leq L\)); \(J_{i,n_i}\) is the precast component produced in production line \(i\) at the sequence \(i\) (\(1 \leq n_i\));
\(J_{i,n_i}\) is the last precast component produced in production line \(i\);
\(M_{i,k}\) is the workstation handling the \(k\)th (\(1 \leq k \leq 6\)) process;
\(S(J_{i,n_i}, M_{i,k})\), \(P_{i,k}\) and \(C(J_{i,n_i}, M_{i,k})\) are the entering time, duration and leaving time of the precast component \(J_{i,n_i}\) in the workstation \(M_{i,k}\) respectively;
\(\text{Max}_{i=1}^{n} f(l)\) is the maximum of \(f(l)\), where \(l\) is a positive integer and \(1 \leq l \leq L\);
\(\tau_{i,j}\) and \(\varepsilon_{i,j}\) are the rates of default penalty and inventory cost of precast component \(J_{i,n_i}\) respectively;
\(d_{i,j}\) is the due date of precast component \(J_{i,n_i}\).

The optimization objective of type continuity of the precast components produced in turn is not included in the existing scheduling models. The inference of it is in the following.

In MP-FSM, the type continuity of the precast components produced in turn in a single production line is guaranteed in two steps. First, the quantity of the types of the precast components in each shift is minimized. For example, if 6 precast components of type A (A for short here after) and 6 precast components of type B (B for short here after) are produced in a single production line, all the As should be assigned to a shift and Bs to another. Second, if the types of the precast components produced in a shift cannot be the same, the ones of the same type are sequenced next to each other. Namely, the quantity of the type changes of precast components produced in turn in each shift is minimized. For instance, 2 As and 1 Bs have to be produced in a single production line in the same shift. Comparing to the sequence of A, B, A with 2 type changes of precast components, those in the sequences of A, A, B or B, A, A are only 1. So the first sequence should be abandoned. It is obvious that the type changes of precast components produced in turn in a single production line are reduced in both the range and the quantity by the two steps.

Based on this analysis, the optimization objectivity is expressed as Equation (4).
\[
\text{Min } f_{\text{TSL}} = \sum_{s=1}^{S} (TN_{i,s} + \alpha \cdot CT_{i,s}) \quad (4)
\]

Where: \( l \) is the number of a production line;
\( s \) is the number of a shift;
\( S \) is the quantity of shifts;
\( TN_{i,s} \) is the quantity of the types of the precast components in the shift \( s \) of the production line \( l \);
\( CT_{i,s} \) is the quantity of the type changes of precast components produced in turn in the shift \( s \) of the production line \( l \);
\( \alpha \) is a positive weight and \( \alpha < 1 \).

\( f_{\text{TSL}} \) represents the degree of the type continuity of the precast components produced in turn in the production line \( l \). The smaller the \( f_{\text{TSL}} \) is, the more continuous the types of the precast components produced in turn are, and vice versa. Therefore, the Equation (4) means that maintaining the type continuity of the precast components produced in turn in a single production line equals minimizing both the quantity of the types and the quantity of the type changes of the precast components produced in turn in each shift.

Based on its definition, the type continuity of the precast components produced in turn in multiple production lines is guaranteed by ensuring that of each single line. So the optimization objectivity is expressed as Equation (5).

\[
\text{Min } f_{\text{TC}} = \sum_{s=1}^{S} \{ETN_{s} + \alpha \cdot ECT_{s}\} \quad (5)
\]

Where: \( ETN_{s} = \sum_{l=1}^{L} \frac{TN_{l,s}^2}{I_{l,s}} \) is the equivalent quantity of the types of the precast components in the shift \( s \) of multiple production lines, where \( I_{l,s} \) the quantity of production lines actually participating in the production in shift \( s \);
\( ECT_{s} = \sum_{l=1}^{L} \frac{CT_{l,s}^2}{I_{l,s}} \) is the equivalent quantity of the type changes of precast components produced in turn in the shift \( s \) of multiple production lines.

\( ETN_{s} \) and \( ECT_{s} \) represent the average quantity of the types and type changes of the precast components in shift \( s \) in the production lines that actually participates in precast production. \( f_{\text{TC}} \) represents the average degree of the type continuity of the precast components produced in turn in multiple production lines. Therefore, the Equation (5) means that maintaining the type continuity of the precast components produced in turn in multiple production lines equals minimizing both the average quantity of the types and the average quantity of the type changes of the precast components produced in turn in multiple production lines in each shift.

Because the optimization objective concerns the experience and efficiency of all the workers, besides optimizing the optimization objective in a global perspective, it is also of significance to avoid the local deterioration of it in certain production lines. Hence, instead of the average of \( TN_{l,s} \) and \( CT_{l,s} \) of each production line, \( ETN_{s} \) and \( ECT_{s} \) are calculated by the average of the squares of them. The reason is explained in the following by taking \( ETN_{s} \) as an example.

If the \( ETN_{s} \) is calculated by averaging the \( TN_{l,s} \) of each production line, namely by \( \sum_{l=1}^{L} \frac{TN_{l,s}}{I_{l,s}} \), \( ETN_{s} \) is not capable to represent the allocation of the types of precast components among lines. For instance, four precast components of type A, B, C, D (A, B, C, D for short hereafter) are produced in two production lines. In plan 1, A is produced in line 1 and B, C, D are produced in line 2. In plan 2, A, B are produced in line 1 and C, D are produced in line 2. The \( \sum_{l=1}^{L} \frac{TN_{l,s}}{I_{l,s}} \) of both plan are 2, but obviously it is easier for workers to make operation mistakes in plan 1. Thus plan 2 should be chosen if the other optimization objectives are not concerned. To solve the problem, this paper calculate the \( ETN_{s} \) by averaging the squares of \( TN_{l,s} \) of each production line, based on the property of the function \( f(a_1, \ldots, a_n) = \sum_{i=1}^{n} a_i^2 / n \) where \( \sum_{i=1}^{n} a_i = \text{constant} \) that the value of the function is minimized when \( a_1 = a_2 \ldots = a_n \). Thus, the possibility that the \( TN_{l,s} \) in certain production lines may be large even...
when the $f_{TC}$ is small is excluded so that the allocation of the types of precast components among production lines is balanced.

In most cases, the rebar cages are made ahead of the flowshop production. Moreover, designers turn to separate the pipes and wires from the concrete components to make the precast production easy. Consequently, the relation of operations in each process and the inner structures of precast components is weak so that maintaining the type continuity of the precast components produced in turn can be simplified as maintaining the their mold type continuity.

### 3.2 Optimization constraints

Optimization constraints in MP-FSM include the constraint of productivity, the constraint of the size of curing rooms, the constraint of the eight-hour day working, the constraint of the buffer size between workstations, the constraint of the quantity of molds and the constraint of the quantity of production pallets corresponding to the optimization constraints listed in Table 1.

The first three optimization constraints of the scheduling models for a single production line have already been presented or introduced in the existing researches (Chan, et al. 2002, Benjaoran, et al. 2005, Ko, et al. 2010). For MP-FSM, only small changes need to be made in the expression. For example, a precast component can be located merely by the sequence of production in a single production line and, thereby, marked as $I_{li}$. Nevertheless, the number of the production line, in which the precast component is produced, is also necessary to locating the precast component in multiple production lines. Hence, the precast components are marked as $I_{li}$ for MP-FSM. For the first three optimization constraints, this paper only presents the equations in the following, as shown in Equations (6) to (11), where Equations (6) and (7) stand for the constraint of productivity; Equation (8) stands for the constraint of the size of curing rooms; Equation (9) stands for the constraint of the eight-hour day working for the casting process; Equation (10) stands for the constraint of the eight-hour day working for the curing process; Equation (11) stands for the constraint of the eight-hour day working for the other processes.

$$S(I_{li}, M_{lk}) \geq \begin{cases} \text{Max}[C(I_{l(i-1)}, M_{lk}), C(I_{li}, M_{l(k-1)})], & \text{if } k \neq 4 \\ C(I_{li}, M_{l(k-1)}), & \text{if } k = 4 \end{cases} \quad (6)$$

$$C(I_{li}, M_{lk}) \geq S(I_{li}, M_{lk}) + P_{l,ik} \quad \text{(7)}$$

$$S(I_{li}, M_{lA}) \geq \text{Max}^{\text{Y}th}_{\text{Y} \leq i} C(I_{lY}, M_{lA}) \quad \text{(8)}$$

$$C(I_{li}, M_{l3}) \geq \begin{cases} T, & \text{if } T \leq 24D + H_W + H_E \\ 24(D + 1) + P_{l,ik}, & \text{if } T > 24D + H_W + H_E \end{cases} \quad \text{(9)}$$

$$C(I_{li}, M_{lA}) \geq \begin{cases} T, & \text{if } T < 24D + H_W \\ 24(D + 1), & \text{if } 24D + H_W \leq T \leq 24(D + 1) \\ T, & \text{if } T > 24(D + 1) \end{cases} \quad \text{(10)}$$

$$C(I_{li}, M_{lk}) \geq \begin{cases} T, & \text{if } T < 24D + H_W \text{ and } k = 1,2,5,6 \\ T + H_N, & \text{if } T \geq 24D + H_W \text{ and } k = 1,2,5,6 \end{cases} \quad \text{(11)}$$

Where: $Y_i$ is the maximum of precast components that can be handled in the curing room of production line $i$;
$\text{Max}^{\text{Y}th}_{\text{Y} \leq i} f(y)$ is the $Y$th maximum value of $f(y)$, where $y$ is a positive integer and $y \leq i$;
$T$ is the $C(I_{li},M_{l,k})$ calculated without considering the constrain of eight-hour day working;
D=integer(T/24) is the number of days passed from the start of the production to the \( C(I_{lj}, M_{lk}) \);

\( H_W, H_N \) and \( H_S \) are the working hours, non-working hours and overtime hours allowed per day.

3.2.1 Constraint of the buffer size

The constraint of the buffer size between workstations has been given by Ko et al. (2010). But errors are found in the given constraint. Due to the space limitation, this paper briefly explains why it is wrong by taking a counter example and, then, presents the correct equation.

The counter example takes place between \( M_{l2} \) and \( M_{l3} \), when \( C(I_{lj}, M_{l1}) > C(I_{lj(i-1)}, M_{l2}) \) and the buffer size \( B_{l2} \) between the workstations is small. Therein, \( B_{l2} \) stands for the maximum quality of precast components that can be stacked between workstation \( M_{l2} \) and \( M_{l3} \). First, assuming that the buffer size between \( M_{l2} \) and \( M_{l3} \) is not considered, the ideal leaving time of \( I_{lj} \) from \( M_{l2} \), namely \( C(I_{lj}, M_{l2}) \), equals \( C(I_{lj}, M_{l1}) + P_{l1,2} \) according to the Equations (6) and (7), which are agreed in both this paper and the existing publications (Ko, et al., 2010). Second, if the ideal leaving time \( C(I_{lj}, M_{l2}) \) is earlier than the entering time of \( I_{lj(i-1),l2} \) into \( M_{l3} \), that is \( C(I_{lj}, M_{l2}) = C(I_{lj}, M_{l1}) + P_{l1,2} < S(I_{lj(i-1)}, M_{l3}) \), more than \( B_{l2} \) precast components are stacked between \( M_{l2} \) and \( M_{l3} \) which exceeds the storage capacity. Then, according to the equations in the existing research, the leaving time of \( I_{lj} \) from \( M_{l2} \), namely \( C'(I_{lj}, M_{l2}) \), is delayed to \( \max\{S(I_{lj(i-1)}, M_{l3}) - [S(I_{lj}, M_{l1}) - C(I_{lj(i-1)}, M_{l2})], C(I_{lj}, M_{l1}) + P_{l1,2}\} \) (Ko, et al., 2010). Nevertheless, it is obvious that \( C'(I_{lj}, M_{l2}) \) is still earlier than \( S(I_{lj(i-1)}, M_{l3}) \), which means the oversaturation problem of the buffer size is not yet resolved.

The essence of the constraint of buffer size is that the space occupied by the work-in-progresses temporarily stacked between two adjacent workstations must be less than the storage capacity between the workstations. Based on the essence, the Equation (12) is established representing the optimization constraint. The meaning of Equation (12) is that the leaving time of the precast component \( I_{lj} \) from the workstation \( M_{lk} \) must be later than the entering time of the precast component \( I_{lj(i)}, M_{lk+1} \) into the workstation \( M_{lk+1} \).

\[
C(I_{lj}, M_{lk}) \geq S(I_{lj}, M_{lk+1})
\]  

(12)

Where: \( B_{lk} \) is the maximum quality of precast components that can be stacked between workstation \( M_{lk} \) and \( M_{lk+1} \).

The constraint of the quantity of molds and production pallets of multiple production lines is introduced in the following.

3.2.2 Constraint of the quantity of molds

As introduced in section 2, it is essential to allocate the molds to the production lines and to ensure the molds of each type are always more than the precast components which are of the same type and being produced in each production line respectively, as show as the item No. 9 of Table 1.

Traditionally, the molds are allocated to each production line by determining the rations. However, the allocation plans of this kind are too complex as an optimization constraint when the rations of molds are time-dependent. MP-FSM allocates the molds by determining the priorities of precast components to use the molds. Namely, each precast component is given a priority. The precast components with higher priorities get molds earlier than the other ones of the same type. If the molds of the type are occupied, the precast components with lower priorities have to wait until any molds of the type turn to be available. In this way, allocation plans are adjusted by changing the priorities of each precast components. More importantly, this paper finds that the ration based molds allocation and the priority based molds allocation are equivalent. The proof is in the following.
In the ration based molds allocation, the ration of the molds of type $s$ at the moment $t$ in the production line $l$ is $Q_{ls}(t)$, while $Q_{ls}(t)$ equals that the original ration $Q_{ls}(t_0)$ at the moment to pluses the increment $\Delta^+Q_{ls}(t)eN$ and minuses the decrement $\Delta^-Q_{ls}(t)eN$ during the period of time $t$, as shown in Equation (13).

$$Q_{ls}(t) = Q_{ls}(t_0) + \Delta^+Q_{ls}(t) - \Delta^-Q_{ls}(t) \quad \text{(13)}$$

The increment of molds $\Delta^+Q_{ls}(t)$ are the most important variable during molds allocation. For one thing, the actual demand of molds $D_{ls}(t)$ in each production line is 1 at the beginning of precast production. If $Q_{ls}(t_0)$ is bigger than 1, extra molds are booked but not utilized. Therefore, an original ration $Q_{ls}(t_0)$ of production line $l$ equals that the ration of the line is only 1 at the beginning but increases rapidly to $Q_{ls}(t_0)$ in a remarkably short period of time. For another, a mold is occupied from the molding process to demolding process of a precast component. So the decrement $\Delta^-Q_{ls}(t)$ is determined by production rules but not schedulers. To sum up, $\Delta^+Q_{ls}(t)$ is the only variable during molds allocation.

Because the decrement of molds $\Delta^+Q_{ls}(t)$ is allocated to the precast components eventually, determining the $\Delta^+Q_{ls}(t)$ of each production line is equivalent to the process, in the perspective of precast components, that the precast productions book the molds according to their priorities.

Furthermore, since molds are not released during the production of precast components, the priorities are constant. Thus, determining the time-dependent rations of molds is transformed into determining the time-independent priorities of precast components.

It is necessary to notice that the priorities of precast components should conform to the production sequence of precast components in each production line. Namely, in a single production line, the earlier the precast component produces, the higher the priority of it should be. Otherwise, the production may be interrupted because all the molds are booked by precast components that are not yet produced. But the precast components produced in the different production lines do not have to comply this rule. For example, if the due date of the precast component at the sequence 3 in the line 1 is earlier than the one at the sequence 2 in the line 2, it is reasonable for the former to enjoy higher priority, although it may be produced later.

As is mentioned before, the precast components with higher priorities get molds earlier than the other ones of the same type, during the priority based molds allocation. So, the constraint of the quantity of molds is expressed as the Equation (14).

$$S \left( J_{ls}, M_{l1} \right) \geq \min \left\{ \max_{y \in N^s | y \leq l \leq x \leq x^*} f(y) \right\} \quad \text{(14)}$$

Where: $J_{ls}$ is the precast component of type $s$ produced in production line $l$ at the sequence, the priority of which is $j$; $Q_s$ is the quantity of molds of type $s$ in the workshop; $\max_{y \in N^s | y \leq l \leq x^*} f(y)$ stands for the first $Q_s$ maximum values of $f(y)$, where $y$ is a positive integer and $y \leq i$.

The meaning of Equation (14) is that any precast components cannot be produced until any of the $s$-type precast components being produced at that time (namely the last $Q_s$ precast components being completed among the $s$-type ones with higher priorities than the precast component waiting to be produced) is completed and releases its mold.

As a typical example of the optimization constraint, five $s$-type precast components of numbered from 1 to 5 are produced in two production lines equipped with 3 molds of the type. The priorities of them to use molds are No. 4> No. 2> No. 5> No. 3> No. 1. The gantt diagram of a feasible flowshop schedule is shown in Figure 1, where the horizontal axis stands for the time and the vertical axis stands for the number of precast components. The colored blocks represent each process of precast production while the numbers inside them represent the production lines in which the processes are handled. As shown in Figure 1, the molds are not
enough at the moment 15h. The precast component 5 occupies the mold released from
precast component 4 in preference to precast component 1, because the priority of precast
component 5 is higher. Thereby, precast component 1 has to wait until the precast component
3 is finished, namely the earliest finish time of the last 3 precast components being completed
among the ones with higher priorities, i.e. precast component 3, 2 and 5.

![Gantt chart of a schedule generated based on MP-FSM](image)

**Figure 1 Gantt chart of a schedule generated based on MP-FSM**

### 3.2.3 Constraint of the quantity of production pallets

As mentioned in section 2, it is essential to allocate the production pallets to the production
lines and to ensure that the production pallets are always more than the precast components
which are being produced in each production line respectively, as shown by the item No. 10
in Table 1.

In MP-FSM, production pallets are allocated according to the aforementioned priorities of
precast components to use molds in order to guarantee that the precast components, which
have already obtained molds, can get production pallets. Unnecessary occupancy of molds
and production pallets is avoided in this way. This paper presents only the equation of the
optimization constraint as shown by Equation (15) due to its similarity to the constraint of the
quantity of molds.

$$S\left( j_{l,1}^l, M_{l,1} \right) \geq \text{Min}\left\{ \text{Max}_{\forall l < l \leq N^*} \{ y_{l,1} + y_{l,2} + y_{l,3} \} \right\} \left\{ C\left( j_{l,1}, y_{l,2}, M_{l,2} \right) \right\}$$

Where: $j_{l,1}^l$ is the precast component produced in production line $l$ at the sequence, the
priority of which is $j$;
P is the quantity of production pallets in the workshop.

The meaning of Equation (15) is that any precast components cannot be produced until
any of the precast components being produced at that time (namely the last $P$ precast
components being completed among the ones with higher priorities than the precast
component waiting to be produced) is completed and releases its mold.

### 4 Basic method for flowshop scheduling based on MP-FSM

The method for obtaining the optimal schedule to a given production arrangement, which
contains allocation plan of the precast components to the production lines, the production
sequence of them and the priorities of them to use molds, is inferred in the following.

First of all, given a production arrangement, each optimization objectivities of MP-FSM can
be realized by ensuring each production process of all the precast components starts and
ends as early as possible. Taking the minimizing of default penalty and inventory cost as an
e.example as shown by the item No. 2 in Table 1. For one thing, in most cases, the inventory
cost is controlled by adjusting the production sequence of precast components to make the
completion time meets the due date. Hence, the inventory cost is fixed as the production
arrangement is given. For another, if each production process of each precast component
starts and ends as early as possible, all the precast components are completed the earliest and the default penalty is the lowest.

Second, all the optimization constraints require that the production processes of all the precast components start and end later than certain moments, as shown in the Equations (6), (7), (8), (9), (10), (11), (12), (14) and (15).

Therefore, if the finished production processes of a precast component and the ones produced before it is given, the start time and the complete time of the next process of it should be the earliest moment that satisfied all the optimization constraints, namely the Equations (6), (7), (8), (9), (10), (11), (12), (14) and (15), in order to realize the optimization objectives. Consequently, the optimal and feasible flowshop schedule of a given production arrangement is obtained by recurrently calculating the start time and the complete time of each process of all the precast components.

According to the method, the steps of flowshop scheduling based on MP-FSM are as following. First, list all the production arrangements of the precast components. Second, calculate the optimal and feasible schedules of all the production arrangements based on the method. Finally, evaluate the schedules by the optimization objectives, namely the Equations (1), (2), (3) and (5), and choose the best one from them. Because the calculation is complex and massive during the steps, artificial intelligence algorithms such as genetic algorithm and taboo algorithm are recommended to be utilized.

5 Conclusion
The research attempts to automatically generate and optimize the flowshop schedules by establishing the flowshop scheduling model of multiple production lines for precast production named MP-FSM, in which pragmatic optimization objectives and constraints are considered. A basic approach for flowshop scheduling of precast production is also proposed based on the MP-FSM. It is concluded that optimal schedules can be obtained by employing the method. This research lays a sound base for developing automatic scheduling systems for precast production in the future.

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