

SIMPLIFIED DESIGN METHOD OF A TRANSMISSION CANAL

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ABSTRACT

A transmission canal loses water through seepage and evaporation. For economy, it should be divided into sub-sections and the cross-section for each of the sub-sections must be designed separately. This adds cost of transition in between two sub-sections, but the transition cost is overcome by reduced cost of the cross-section. Optimal design parameters for transmission canal based on the Manning equation are not available yet. This paper presents design equations for the least cost transmission canal considering earthwork cost which may vary with depth of excavation, cost of lining, and cost of water lost as seepage and evaporation from irrigation canals of triangular, rectangular, and trapezoidal shapes. This optimization problem is some sort of a dynamic programming, which is complicated due to unknown number of subsections i.e. number of unknown constraints. The problem was expressed in dimensionless form and then solved numerically. The optimal design equations along with the tabulated section shape coefficients provide a convenient method for the optimal design of a transmission canal. These optimal design equations and coefficients have been obtained by analyzing a very large number of optimal sections resulted from application of optimization procedure in the wide application ranges of input variables. The analysis consists of conceiving an appropriate functional form and then minimizing errors between the optimal values and the computed values from the conceived function with coefficients. Using the proposed equations along with the tabulated section shape coefficients, the optimal number of subsections and corresponding cost of a transmission canal can be obtained in single step computations.

KEY WORDS

Canals, canal design, optimal sections, transmission canal, minimum cost section.

INTRODUCTION

Canals continue to be major conveyance systems for delivering water for irrigation. The seepage loss from irrigation canals constitutes a substantial percentage of the usable water (Rohwer and Stout 1948). According to the Indian Standard ("Bureau" 1980) the loss of water by the seepage from unlined canals in India generally varies from 0.3 to 7.0 m³/s per 10⁶ m² of wetted surface. Canals are lined to check the seepage. But canal lining deteriorates with time and hence, significant seepage losses continue to occur from a lined canal

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(Wachyan and Rushton 1987). A transmission canal conveys water from the source to a distribution canal. Many a times the area to be irrigated lies very far from the source, hence requires long transmission canals e.g. the Rajasthan canal system has the transmission canal length of 204 km carrying a discharge about 524 m³/s (Hooja 1993; Kanwar Sain 1967). Though, there is no withdrawal from a transmission canal but it loses water through seepage and evaporation.

Since, a transmission canal loses water through seepage and evaporation, it is not economical to continue the same section throughout the length of a long transmission canal. Instead the transmission canal should be divided into sub-sections or reaches and the cross-section for each of the sub-sections must be designed separately. This adds cost of transition in between two sub-sections, but the transition cost is overcome by reduced cost of the cross-section. The reduced cross-section not only results into cost saving for earthwork, lining and water lost, but also requires less cost in land acquisition, in construction of bridges and cross-drainage works. Though the author's team (Chahar 2000, Swamee et al 2002b) proposed optimal design equations for transmission canal using general resistance equation based on Darcy-Weisbach friction formula and Colebrook formula but such equations based on the Manning equation are not available yet. This paper presents design equations for optimal transmission canals based on Manning's equation.

WATER LOSS FROM A CANAL

Chahar (2000) and Swamee et al (2002a) expressed the quantity of water loss q_w (m²/s) as seepage q_s (m²/s) and evaporation q_E (m²/s) from a unit length of canal as

$$q_w = q_s + q_E = kF_s y_n + ET \quad (1)$$

where k = hydraulic conductivity of the porous medium (m/s); F_s = seepage function (dimensionless), which is a function of channel geometry and depth to drainage layer below the canal; y_n = normal depth of flow (m); E = evaporation discharge per unit surface area (m/s); and T = width of free surface (m). Simple algebraic equations for the seepage function as given by Chahar (2000) and Swamee et al. (2001b) for triangular, rectangular, and trapezoidal canal sections underlain by a drainage layer at depth d are

$$F_s = \left\{ \left(\frac{1.81m^{1.18} + 2.1}{(d/y - 1)^{0.26}} \right)^{9.35} + \left[(4\pi - \pi^2)^{1.3} + (2m)^{1.3} \right]^{7.2} \right\}^{0.107} \quad (2)$$

$$F_s = \left\{ \left(\frac{2.5(b/y_n)^{0.84} + 0.45}{(d/y_n - 1)^{0.69}} \right)^{2.38} + \left[(4\pi - \pi^2)^{0.77} + (b/y_n)^{0.77} \right]^{3.094} \right\}^{0.42} \quad (3)$$

and

$$F_s = \left\{ \left(1.81(m^{1.3} + 1.432(b/y_n)^{0.93})^{0.9} + \frac{b + 100my_n}{2.22b + 47.62my_n + 1.57bm^5} \right)^{p_1} \left(\frac{d}{y_n} - 1 \right)^{-p_1 p_2} + \left((4\pi - \pi^2)^{1.3} + (2m)^{1.3} \right)^{0.77 p_3} + (b/y_n)^{p_3} \right\}^{\frac{1}{p_3}} \quad (4)$$

respectively. Where $p_1 = \frac{2.38b + 7.48my_n}{b + 0.8my_n}$; $p_2 = \frac{0.318b + 0.26my_n}{0.461b + my_n}$; $p_3 = \frac{1 + 0.6m}{1.3 + 0.6m}$; $b =$ bed width (m); and $m =$ side slope (1 horizontal in m vertical). As $d \rightarrow \infty$, (2) to (4) becomes function of canal geometry only and reduces to the seepage functions for canals passing through a homogeneous porous medium of very large depth (Swamee et al 2000b).

UNIT LENGTH CANAL SECTION COST

The most general case for the optimal channel design is that which considers cost of earthwork per unit length of canal C_e (\$/m) that varies with depth of the canal, cost of lining per unit length of canal C_L (\$/m), and capitalized cost of water lost as seepage and evaporation per unit length of canal C_w (\$/m). The cost of canal per unit length C (\$/m) was obtained (Chahar 2000; Swamee et al 2000c) as

$$C = C_e + C_L + C_w = c_e A + c_r A \bar{y} + c_L P + c_{ws} F_s y_n + c_{wE} T \quad (5)$$

where $c_e =$ cost per unit volume of earthwork at ground level (\$/m³); $c_r =$ increase in the unit excavation cost per unit depth (\$/m⁴); $c_L =$ cost per unit surface area of lining (\$/m²); $c_{ws} = 3.156 \times 10^7 c_w k / r$ (\$/m²); $c_{wE} = 3.156 \times 10^7 c_w E / r$ (\$/m²); $c_w =$ cost per unit volume of water (\$/m³); $r =$ rate of interest (\$/\$/year); and $\bar{y} =$ depth of centroid of excavated area from the free water surface (m).

It can be seen from (5) that for given cost factors c_e , c_r , c_L , c_{ws} , and c_{wE} the cost per unit length of the canal is a function of the canal geometry and depth of the drainage layer, since d appears in the seepage function. As c_L / c_e , c_e / c_r , c_{ws} / c_e , and c_{wE} / c_e have length dimension, they remain unaffected by the monetary units chosen. These ratios can be obtained for various types of linings, soil strata, and climatic condition by using appropriate unit rates (Swamee 2000c).

REQUIREMENTS FOR FLOW IN A CANAL

A rigid boundary canal is designed for the condition of uniform flow. The most commonly used uniform flow formula around the world is the Manning equation (Chow 1973) due to its simplicity and acceptable degree of accuracy in most of practical applications. The uniform flow rate or discharge Q (m³/s) in a canal by Manning's equation is

$$Q = AV = \frac{1}{n} AR^{2/3} S_f^{1/2} = \frac{1}{n} A \left(\frac{A}{P} \right)^{2/3} S_f^{1/2} = \frac{1}{n} A \left(\frac{A}{P} \right)^{2/3} S_o^{1/2} \quad (6)$$

where V = mean velocity of uniform flow (m/s); R = hydraulic radius (m), defined as the ratio of flow area A (m^2) to the flow perimeter P (m); n = Manning's roughness coefficient; S_f = energy slope (dimensionless); and S_o = bed slope of the canal (dimensionless). For uniform flow $S_f = S_o$. In the Manning's formula all the terms except n can be directly measured. The roughness coefficient is a parameter representing the integrated effects of the channel cross-sectional resistance. The selection of a value of n is subjective, based on experience and engineering judgement. Chow (1973) lists values of n for different conditions of a canal.

Since the least cost canal section is designed to sustain uniform flow, (6) provides the required condition as an equality constraint function in the design.

METHODOLOGY

PROBLEM FORMULATION

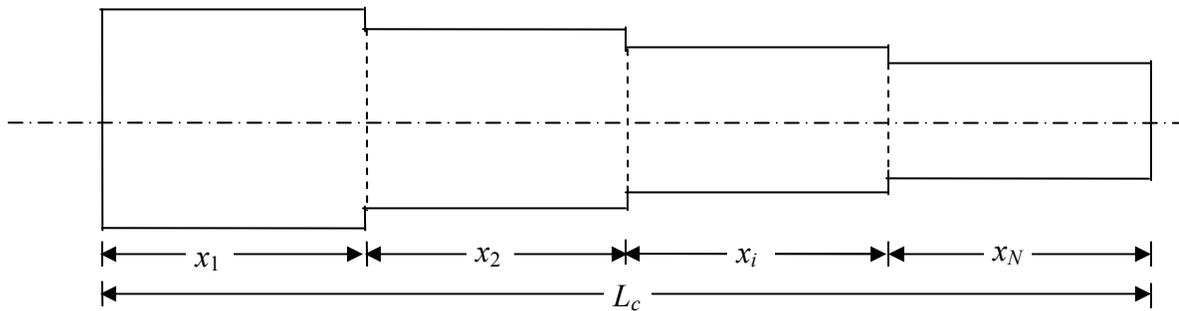


Figure 1: Layout of a Transmission Canal

A transmission canal of length L_c (m) was divided into N subsections of $x_1, x_2, x_3, \dots, x_N$ lengths (m). Hence, $N - 1$ number of transitions were required in the transmission canal. The cost of a transmission canal consists of the cost of subsections and the cost of transitions. Letting the cost of each transition C_T (monetary units, \$) same for all the transitions, and assuming canal geometry and depth to the drainage layer remains constant throughout the each subsection, the overall cost of the transmission canal C_o (monetary units, \$) would be

$$C_o = \sum_{i=1}^N C_i x_i + (N - 1)C_T = \sum_{i=1}^N (c_e A_i + c_r A_i \bar{y}_i + c_L P_i + c_{ws} y_{ni} F_{si} + c_{wE} T_i) x_i + (N - 1)C_T \quad (7)$$

Therefore, the problem of the least cost design of a transmission canal became

$$\text{Minimize } C_o = \sum_{i=1}^N (c_e A_i + c_r A_i \bar{y}_i + c_L P_i + c_{ws} y_{ni} F_{si} + c_{wE} T_i) x_i + (N-1) C_T \quad (8)$$

Subject to

$$Q_i - \frac{1}{n} \frac{A_i^{5/3}}{P_i^{2/3}} S_0^{1/2} = 0 \quad \text{for all } i \quad (9a)$$

$$Q_{i+1} - Q_i + (k_i y_{ni} F_{si} + E_i T_i) x_i = 0 \quad \text{for all } i \quad (9b)$$

$$\sum_{i=1}^N x_i - L_c = 0 \quad (9c)$$

where the index i indicates a sub-section.

The first constraint (9a) imposes the condition of uniform flow in the each subsection, while, the second constraint (9b) satisfies the continuity in discharge from one section to the next section, and the third constraint (9c) is obvious. The optimization problem stated by (8) and (9a-c) is some sort of a dynamic programming. The problem is complicated due to unknown number of subsections N i.e. number of unknown constraints. This optimization problem was expressed in dimensionless form and then solved.

NON-DIMENSIONALISATION

Assuming S_0 , n , k , and E constant for all the subsections and using a length scale λ (m) as

$$\lambda_i = (Q_i n / \sqrt{S_0})^{3/8} \quad (10)$$

and $\lambda = \lambda_1$; the following dimensionless parameters were obtained

$$C_{o*} = C_o / (c_e \lambda^2 L_c); \quad c_{r*} = c_r \lambda / c_e; \quad c_{L*} = c_L / (c_e \lambda) \quad (11a-c)$$

$$c_{ws*} = c_{ws} / (c_e \lambda); \quad c_{wE*} = c_{wE} / (c_e \lambda); \quad c_{T*} = C_T / (c_e \lambda^2 L_c) \quad (11d-f)$$

$$A_{i*} = A_i / \lambda_i^2; \quad \bar{y}_{i*} = \bar{y}_i / \lambda_i; \quad P_{i*} = P_i / \lambda_i \quad (11g-i)$$

$$y_{ni*} = y_{ni} / \lambda_i; \quad T_{i*} = T_i / \lambda_i; \quad R_{i*} = R_i / \lambda_i \quad (11j-l)$$

$$x_{i*} = x_i / L_c; \quad k_* = k L_c / \sqrt{g S_0 \lambda^3}; \quad E_* = E / k; \quad \lambda_{i*} = \lambda_i / \lambda \quad (11m-p)$$

where variables with subscript * denotes corresponding non dimensional parameter. Using (11a-p) in (8) and (9a-c), the optimization problem in non-dimensional form reduced to:

Minimize

$$C_{o*} = \sum_{i=1}^N (A_{i*} \lambda_{i*}^2 + c_{r*} A_{i*} \bar{y}_{i*} \lambda_{i*}^3 + c_{L*} P_{i*} \lambda_{i*} + c_{ws*} y_{ni*} F_{si} \lambda_{i*} + c_{wE*} T_{i*} \lambda_{i*}) x_{i*} + (N-1) c_{T*} \quad (12)$$

Subject to

$$\Phi_i = A_{i^*}^{5/3} - P_{i^*}^{2/3} = A_{i^*}^5 - P_{i^*}^2 = 0 \quad \text{for all } i \quad (13a)$$

$$\lambda_{(i+1)^*}^{2.5} - \lambda_{i^*}^{2.5} + k_*(y_{ni^*}F_{si} + E_*T_{i^*})x_{i^*} = 0 \quad \text{for all } i \quad (13b)$$

$$\sum_{i=1}^N x_{i^*} - 1 = 0 \quad (13c)$$

This dimensionless optimization problem was simplified to an optimization problem with one variable N . This was achieved by providing the least cost section for each subsection of the transmission canal, and considering the length of the each subsection to be equal.

LEAST COST SECTION DESIGN EQUATIONS FOR EACH SUBSECTION

The cost of the each subsection must be minimum to arrive at the least cost transmission canal. The least cost canal section for a particular subsection of the transmission canal could be obtained from (8), without the transition cost, subject to (9a) and dropping the index i from them. The optimization problem in dimensionless form became

$$\text{Minimize } C_* = A_* + c_{r^*}A_*\bar{y}_* + c_{L^*}P_* + c_{ws^*}F_s y_{n^*} + c_{wE^*}T_* \quad (14)$$

$$\text{Subject to } \Phi = A_*^5 - P_*^2 = 0 \quad (15)$$

This nonlinear optimization problem with equality constraint was numerically solved for triangular, rectangular, and trapezoidal channel sections using the procedure similar to Swamee et al (2000a-c; 2001a,b; 2002a,b) and Chahar (2000).

OPTIMAL COST AND LENGTH OF EACH SUBSECTION

Use of the optimal side slope, bed width, and normal depth as obtained above for designing each subsection automatically satisfies the constraint (13a). The optimization problem was further simplified by assuming the length of the each sub-section of the transmission canal to be same i.e.

$$x_i = x = L_c / N \quad \text{or} \quad x_{i^*} = x_* = 1 / N \quad \text{for all } i \quad (16)$$

Thus two constraints were eliminated and the problem became

Minimize

$$C_{o^*} = \frac{1}{N} \sum_{i=1}^N (A_{i^*}\lambda_{i^*}^2 + c_{r^*}A_{i^*}\bar{y}_{i^*}\lambda_{i^*}^3 + c_{L^*}P_{i^*}\lambda_{i^*} + c_{ws^*}y_{ni^*}F_{si}\lambda_{i^*} + c_{wE^*}T_{i^*}\lambda_{i^*}) + (N-1)c_{T^*} \quad (17)$$

$$\text{Subject to } \lambda_{(i+1)^*}^{2.5} - \lambda_{i^*}^{2.5} + k_*(y_{ni^*}F_{si} + E_*T_{i^*})/N = 0 \quad \text{for all } i \quad (18)$$

Once the optimal dimensions of a section are fixed using (17), the discharge or λ for the next section becomes a function of N satisfying (18). Now the optimization problem stated by (17) and (18) is left with finding the minimum of (17) for only one unknown N . By

applying Fibonacci search (Bazaraa and Shetty 1979) on triangular, rectangular, and trapezoidal canal sections, a large number of optimal N were obtained for a number of input variables varying in the ranges

$$10^{-5} \leq c_{T^*} \leq \infty; \quad 0 \leq k_* \leq 1.0 \quad (19)$$

Analysis of the optimal data so obtained resulted to the following equation for optimal number of sub-sections in the transmission canal for all the three canal shapes:

$$N^* = L_c \sqrt{\left(\frac{k + k_{Nd0} k \lambda / d + k_{NE0} E}{\sqrt{g \lambda S_0}} \right) \left(\frac{c_e \lambda + k_{Nr} c_r \lambda^2 + k_{NL} c_L + k_{Ns} (1 + k_{Nd} \lambda / d) L_s c_w + k_{NE} L_E c_w}{k_{NT} C_T} \right)} \quad (20)$$

where the subscripts N , and T in section shape coefficients denote number of sub-sections, and transition, respectively. The value obtained from (20) to be rounded to nearest integer. If the optimal number of sub-sections happens to be zero or one then no transition is required and assume $N^* = 1$. The optimal length of the each sub-section x^* of the transmission canal was given by

$$x^* = L_c / N^* \quad (21)$$

Further analysis of the optimal costs resulted to an empirical equation for the minimum cost of the transmission canal as given below:

$$C_o^* = C^* L_c \left[1 + \left(\frac{k_{Tc} L_c}{\lambda} \frac{(k + k_{Td0} k \lambda / d + k_{TE0} E)}{\sqrt{g \lambda S_0}} \right)^{1.28} \left(\frac{c_e \lambda^2 + k_{Tr} c_r \lambda^3 + k_{TL} c_L \lambda + k_{Ts} (1 + k_{Td} \lambda / d) L_s c_w \lambda + k_{TE} L_E c_w \lambda}{k_{Tc1} C_T / L_c + c_e \lambda^2 + k_{Tr1} c_r \lambda^3 + k_{TL1} c_L \lambda + k_{Ts1} L_s c_w \lambda + k_{TE1} L_E c_w \lambda} \right)^{-1} \right] \quad (22)$$

For a large depth of the drainage layer or water table and negligible evaporation loss (21) and (22) reduces to the following simplified expressions:

$$x^* = \sqrt{\left(\frac{\sqrt{g \lambda S_0}}{k} \right) \left(\frac{k_{NT} C_T}{c_e \lambda + k_{Nr} c_r \lambda^2 + k_{NL} c_L + k_{Ns} L_s c_w} \right)} \quad (23)$$

$$C_o^* = C^* L_c \left[1 + \left(\frac{k_{Tc} L_c}{\lambda} \frac{k}{\sqrt{g \lambda S_0}} \right)^{1.28} \left(\frac{c_e \lambda^2 + k_{Tr} c_r \lambda^3 + k_{TL} c_L \lambda + k_{Ts} L_s c_w \lambda}{k_{Tc1} C_T / L_c + c_e \lambda^2 + k_{Tr1} c_r \lambda^3 + k_{TL1} c_L \lambda + k_{Ts1} L_s c_w \lambda} \right)^{-1} \right] \quad (24)$$

Table 1: Coefficients for a Transmission Canal

Entity	Coefficients	Section Shape		
		Triangular	Rectangular	Trapezoidal
(1)	(2)	(3)	(4)	(5)
Transition	k_{NT}	8.72	8.72	9.75
	k_{Tc}	0.109	0.111	0.105
	k_{Tc1}	37.76	37.16	40.92
Earthwork	k_{Nr}	0.388	0.496	0.662
	k_{Tr}	0.556	0.546	0.503
	k_{Tr1}	0.466	0.496	0.45
Lining	k_{NL}	1.09	1.09	0.79
	k_{TL}	0.727	0.727	0.7
	k_{TL1}	1.544	1.562	1.486
Seepage	k_{Nd0}	0.05	0.14	0.15
	k_{Ns}	1.46	1.53	1.49
	k_{Nd}	0.17	0.20	0.21
	k_{Td0}	0.10	0.12	0.15
	k_{Ts}	1.46	1.53	1.25
	k_{Td}	0.24	0.17	0.16
	k_{Ts1}	3.085	3.137	2.615
Evaporation	k_{NE0}	0.47	0.35	0.40
	k_{NE}	0.86	0.57	0.85
	k_{TE0}	0.55	0.37	0.48
	k_{TE}	0.128	0.073	0.352
	k_{TE1}	0.09	0.00	0.57

Similarly other cases can be reduced from (20) and (22). Coefficients for transmission canal appearing in (20) and (22) are listed in Table 1. These optimal design equations and coefficients have been obtained by analyzing a very large number of optimal sections resulted from application of optimization procedure in the wide application ranges of input variables. The analysis consists of conceiving an appropriate functional form and then minimizing errors between the optimal values and the computed values from the conceived function with coefficients. Direct optimization procedures may be adopted for the optimal design of irrigation canal sections and for the transmission canal but they are of limited use and require considerable amount of programming and computation. On the other hand, using the optimal

design equations along with the tabulated section shape coefficients, the optimal design variables of a transmission canal can be obtained in single step computations. Equation (20) along with (21) shows that the optimal subsection length of the transmission canal is independent of the length of the transmission canal. Further, x^* increases with increase in C_T and depth of the drainage layer, while it decreases with increase in hydraulic conductivity of the porous medium, rate of evaporation, earthwork cost, lining cost and cost of water as expected. The optimal cost of the transmission canal is less sensitive to the optimal number of subsections. Though the optimal design of the transmission canal is obtained with assumption of equal cost for each of the transitions and of equal length for each of the subsections of the transmission canal, the method can be extended for unequal cost of transitions and unequal length of subsections. The assumption of equal length of subsection in obtaining the optimal length of a subsection from (21) can be relaxed by obtaining a new x^* each time for the remaining section of the transmission canal.

CONCLUSIONS

Generalized explicit equations have been presented for the optimal number of subsections and corresponding cost of a transmission canal. Using the proposed equations along with the tabulated section shape coefficients, the optimal parameters can be obtained in single step computations for transmission canals of triangular, rectangular, and trapezoidal shapes. Direct optimization procedures may be adopted for the optimal design of the transmission canal but they require considerable amount of programming and computation. The method can be extended to find the coefficients in the optimal design equations for other shapes such as circular section, parabolic section, rounded corner trapezoidal section, etc. if the corresponding seepage functions are developed. Furthermore, the present method can be extended in developing equations for the optimal design of a transmission canal having unequal cost of transitions and unequal length of subsections.

REFERENCES

- Bazaraa, M.S., and Shetty, C.M. (1979). *Nonlinear programming: theory and algorithms*. John Wiley & Sons, Inc., New York.
- Bureau of Indian Standards. (1980). "Measurement of Seepage Losses from Canals." *IS:9452* (part I & II), New Delhi.
- Chahar, B.R. (2000). Optimal design of channel sections considering seepage and evaporation losses. *Ph. D. Thesis Submitted to Dept. of Civil Engrg.*, Univ. of Roorkee, Roorkee, India.
- Chow, V.T. (1973). *Open channel hydraulics*. McGraw-Hill, New York.
- Hooja, R. (1993). "The Indira Gandhi canal project in brief." *J. Irrig. and Power*, Central Board of Irrig. And Power, New Delhi, 50(4), 9-20.
- Kanwar Sain (1967). "Lining of Rajasthan canal." *Pub. No. 82, Symposium on Canal Lining*, Central Board of Irrig. And Power, New Delhi, 152-171.
- Rohwer, C., and Stout, O.V.P. (1948). *Seepage losses from irrigation canals*. Tech. Bul. 38, Colorado Agricultural Experiment Station, Fort Collins.

- Swamee, P.K., Mishra, G.C., and Chahar, B.R. (2000a). "Minimum cost design of lined canal sections." *J. Water Resour. Mangmnt.*, 14(1), 1-12.
- Swamee, P.K., Mishra, G.C., and Chahar, B.R. (2000b). "Design of minimum seepage loss canal sections." *J. Irrig. and Drain. Engrg., ASCE*, 126(1), 28-32.
- Swamee, P.K., Mishra, G.C., and Chahar, B.R. (2000c). "Comprehensive design of minimum cost irrigation canal sections." *J. Irrig. and Drain. Engrg., ASCE*, 126(6), 322-327.
- Swamee, P.K., Mishra, G.C., and Chahar, B.R. (2001a). "Design of minimum earthwork cost canal sections." *J. Water Resour. Mangmnt.*, 15(1), 17-30.
- Swamee, P.K., Mishra, G.C., and Chahar, B.R. (2001b). "Design of minimum seepage loss canal sections with drainage layer at shallow depth." *J. Irrig. and Drain. Engrg., ASCE*, 127(5), 287 – 294.
- Swamee, P.K., Mishra, G.C., and Chahar, B.R. (2002a). "Design of minimum water-loss canal sections." *J. Hyd. Res., IAHR*, 40(2), 215-220.
- Swamee, P.K., Mishra, G.C., and Chahar, B.R. (2002b). "Optimal design of transmission canal." *J. Irrig. and Drain. Engrg., ASCE*, 128(4), 234 – 243.
- Wachyan, E., and Rushton K.R. (1987). "Water losses from irrigation canals." *J. Hydrology*, 92(3-4), 275-288.