

# A CA AND ANN TECHNIQUE OF PREDICTING FAILURE LOAD AND FAILURE PATTERN OF LATERALLY LOADED MASONRY PANEL

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## Abstract

In spite of more than 30 years of research, it is difficult to reliably predict the behaviour of masonry panel subjected to lateral loading. This paper introduces an innovative technique, which combines the cellular automata (CA) and artificial neural networks (ANN) to predict both failure loads and failure patterns of laterally loaded masonry panels without using conventional techniques.

This paper proposes a new concept, called *generalized panel*, which can project various failure patterns of panels into one panel. In other words, these numerical patterns of the panels are mapped on a generalized panel that symbolizes the failure patterns for the panels having different shapes, sizes and boundary conditions. The data are then transferred as the input for the proposed ANN model. The output from the ANN model is the failure load values of the panels. Finally, the ANN model relates the failure load with the corresponding failure pattern of a panel, and subsequently predicts the failure load of the panel.

## KEY WORDS

failure load, failure pattern, artificial neural networks, cellular automata, *generalized panel*, *zone similarity*.

## Introduction

In the past numerical analytical tools have been widely used to predict the failure load and the corresponding failure pattern of a panel. The accuracy of numerical analytical models mostly depends on modeling the constitutive relationship for material more accurately. One of the most difficult research subjects is to model the constitutive relationship of masonry and the behaviour of laterally loaded masonry wall panels. This is because masonry is a highly anisotropic composite material.

Researchers have proposed three typical analytical methods. Baker (1982) and Chong (1993) statistically established the smeared material properties for brickwork by testing a

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number of wallets, and applied the cracking criteria related to flexural tensile stress in their finite element analysis (FEA) of laterally loaded masonry panels. Lee et al. (1996) introduced a two-stage of homogenization technique to investigate the elastic-brittle behaviour of masonry panels subjected to lateral loading; one stage for the orthotropic material; and the other for smeared cracking of the material. Lourenco (1997, 2000) proposed an anisotropic softening model so that a process was made for predicting the response of masonry panels subjected to out-of-plane loading. The above three models do not involve modelling variation of masonry properties in local regions within the panel, which considerably affects the accuracy of the existing analytical techniques.

However, the existing analytical techniques on masonry structures, such as the FEA methods based on the above constitutive models, have shown that no matter how accurate the constitutive models for masonry properties are, they can not predict the failure load and failure pattern of the masonry structure accurately as long as they use global material properties for the whole structure. Lawrence (1991) indicated that “the greatest difficulty with analyzing walls under lateral loading is coping with the high degree of random variation present in masonry materials. ... It is essential to account for this inherent random variation in any theoretical analysis.” In other words, it is impossible to make an accurate prediction if the variation of masonry properties in the local working environment of a masonry structure is not included in the analytical techniques. Therefore, some researchers sought the analytical techniques which can quantify the variation of masonry properties at locations within the masonry structure and used the intelligent techniques, such as neural networks and cellular automata, to deal with predicting the response of masonry panels under lateral loading.

Lawrence (Lawrence and Lu 1991) introduced random noise to tensile strengths at various locations on the panel in order to improve the accuracy of the FEA of masonry panels, but in some cases this approach leads to less accurate prediction of cracking load. Mathew et al. (1999) introduced a hybrid system which combines both case-based reasoning and the artificial neural networks (ANN) based analysis to predict the failure load of masonry wall panels under lateral loading. But for some particular sets of parameters involving in structural configurations, boundary conditions and material properties, the trained ANN can not distinguish their differences well and results in that some predicted failure loads have the same value.

Zhou et al (Zhou, 2002; Rafiq et al, 2003) reported that the panel response to lateral loading is influenced by the variation in masonry properties and most importantly by the boundary. They introduced a new concept “the concept of stiffness/strength corrector” to quantify the variation in masonry properties and the effect of panel boundaries. Then they introduced the concept of zone similarity is basically based on the assumption that the corrector values are governed by the relative distances of a zone from similar boundary types. Based on this concept of zone similarity, a technique using Cellular Automata (CA) was developed to establish corrector values for any unseen panels. Rafiq et al (2006) has extended and refined the concept of stiffness/strength corrector to obtain a close match between load-deflection and failure behavior of various panels. Results from a non-linear FEA demonstrated that using corrector values can greatly improve the prediction of both failure load and failure pattern of the panel.

Zhou and Rafiq et al (2006) further found that the CA model and corresponding criteria for matching zone similarity can be directly used to predict the failure pattern of unseen panels based on the failure pattern of the base panel.

This paper relates the predicted failure pattern of the corresponding failure load of the panel using a new concept of *generalized panel*. In this approach, the ANN model of the panel is used to predict the failure load of the panel based on the predicted failure pattern, obtained by the above CA model. Thus an artificial intelligent technique (AIT) is implemented to directly predict both the failure load and failure pattern of the panel subject to lateral loading without using any conventional methods such as the FEA.

## The CA modelling of boundary effect on zones within a pnel

### The CA model properties

Edward (1989) describes CA as discrete space-time models consisting of cells in a lattice network. The “neighborhood” consists of adjacent cells which will influence the behaviour of a particular cell state (Soschinske 1997). Fig. 1 shows a typical 2-D neighborhood cell model developed by von Neumann (Soschinske 1997) and utilised in this paper.

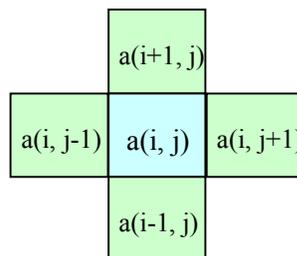


FIG. 2. Cellular Automata Neighbourhoods of von Neumann (1997)

Halpern et al. (1989) formalised the CA transition model in the case of the von Neumann neighbourhood as:

$$a_{i,j}^{(t+1)} = f(a_{i,j}^{(t)}, a_{i,j+1}^{(t)}, a_{i+1,j}^{(t)}, a_{i,j-1}^{(t)}, a_{i-1,j}^{(t)}) \quad (1)$$

where  $a$  = cell state value at a given time interval  $t$ ,  $i, j = x, y$  cell coordinates,  $t$  = time interval, and  $f$  = transition function describing iteration rule.

Rucker and Rudy (1989) summarised the properties of the CA as follows:

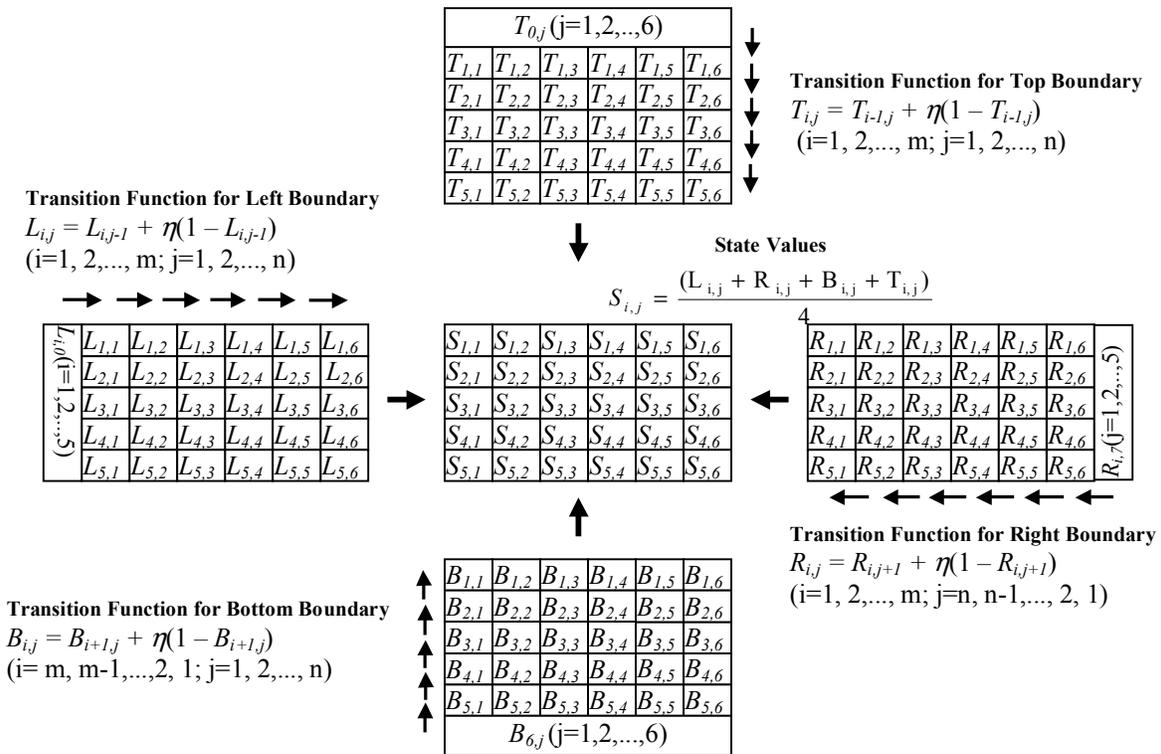
**Parallel:** each cell is updated independent of other cells;

**Locality:** the state value of a new cell depends on its old cell state value, and the values of its neighbourhood cells at a given time  $t$ ;

**Homogeneity:** the same rules are applied to all cells.

### The CA modelling of boundary effect on zones within a panel

Boundary effect on zones (Zhou and Rafiq et al, 2002, 2003) within panels with similar properties can be suitably described by the CA space properties, parallel, locality and homogeneity. Fig. 3 shows how the boundary effect is modelled using CA:



**FIG. 3.** The CA Model of Boundary Effect on Zones within a Panel

- The panel is divided into a number of zones (cells in a CA model). Boundaries of the panel are described as the specified input initial values of the transition functions which are defined in Eqn (2) below. The position of each cell in the CA model corresponds to the position of a zone within the panel.
- Each cell (zone) receives the boundary effect from its neighborhood cells and in turn propagates the boundary effect to their neighborhood cells. For a two-dimensional panel, the von Neumann model was found to be sufficient for describing the effect of different boundaries from four supports at edges of the panel. Therefore, the CA transition functions, which are defined in Eqn. (2), fully propagate the effect of panel boundaries to individual zones within the panel.

$$\begin{aligned}
 L_{i,j} &= L_{i,j-1} + \eta(1 - L_{i,j-1}) \quad (i = 1, 2, \dots, M; j = 1, 2, \dots, N) \\
 R_{i,j} &= R_{i,j+1} + \eta(1 - R_{i,j+1}) \quad (i = 1, 2, \dots, M; j = N, N-1, \dots, 2, 1) \\
 B_{i,j} &= B_{i+1,j} + \eta(1 - B_{i+1,j}) \quad (i = M, M-1, \dots, 2, 1; j = 1, 2, \dots, N) \\
 T_{i,j} &= T_{i-1,j} + \eta(1 - T_{i-1,j}) \quad (i = 1, 2, \dots, M; j = 1, 2, \dots, M)
 \end{aligned} \tag{2}$$

where  $M$  and  $N$  are the numbers of columns and rows of divided zones,  $\eta$  is the coefficient of transition, and  $L$ ,  $R$ ,  $B$  and  $T$  are the state values of zones (cells) which are obtained from the propagation of the collective effect from the left, right, bottom and top boundaries respectively.

$L_{i,0}$ ,  $R_{i,N+1}$ ,  $B_{M+1,j}$  and  $T_{0,j}$  are the input initial values for the transition functions  $L_{i,j}$ ,  $R_{i,j}$ ,  $B_{i,j}$  and  $T_{i,j}$ . These initial values describe different boundary types identified by specific values, for instance, 0.0 defines a free support, 0.2 defines a simple support and 0.4 defines a built-in support. For more details of selecting initial values, refer to Zhou (2002) and Rafiq et al. (2003).

The state value  $S_{i,j}$  of every zone within the panel is calculated as the average effect from its four adjacent cells, see Eqn (3), which shows that the state value for each cell is closely related to its four neighbourhoods.

$$S_{i,j} = \frac{(L_{i,j} + R_{i,j} + B_{i,j} + T_{i,j})}{4} \quad (i = 1, 2, \dots, M, j = 1, 2, \dots, N) \tag{3}$$

The above proposed CA modelling of boundary effects on zones within the panel reflects the CA properties of parallel, locality and homogeneity. For the property of parallel: the state values of individual cells can be updated independent of other cells/zones to assign unique values to each zone using Eqns (2) and (3). For the property of locality: the new cell/zone state value depends on state values of its neighbouring cells/zones (Eqn 3). For the property of homogeneity: the same rules, Eqns (2) and (3), are applied to all cells/zones within the panel.

### criteria for matching zone similarity

The concept of zone similarity (Zhou and Rafiq et al, 2002, 2003) identifies zones within two panels, which are governed by similar boundary types. For this purpose, a criterion needs to be established to match similar zones between a new panel (unseen panel) and a base panel (panel for which the correctors are known) based on the concept of zone similarity. The general criterion for matching zone similarity in the above CA model can be defined as:

Firstly, using the Eqns (4) and (5), calculate state values for all zones within the base panel and the new panel:

$$\{S_i^{new}\} = \{f(B_l, B_r, B_b, B_t; d_{li}, d_{ri}, d_{bi}, d_{ti})\} \tag{4}$$

$$\{S_k^{base}\} = \{f(B'_l, B'_r, B'_b, B'_t; d'_{lk}, d'_{rk}, d'_{bk}, d'_{tk})\} \tag{5}$$

where

$f$  is the function relationship.

$\{S_i^{new}\}$  is the state vector related to the zone  $i$  on the new panel, which includes the state values of this zone itself and its four neighbourhoods.

$\{S_k^{base}\}$  is the state vector related to the zone  $k$  on the base panel, which includes the state values of this zone itself and its four neighbourhoods.

$B_l, B_r, B_b, B_t$  are the boundary type parameters at the left, right, bottom and top edge of the new panel respectively.

$B'_l, B'_r, B'_b, B'_t$  are the boundary type parameters at the left, right, bottom and top edge of the base panel separately.

$d_{li}, d_{ri}, d_{bi}, d_{ti}$  are the distances from the centre of the zone  $i$  to the left, right, bottom and top edges of the new panel respectively.

$d'_{lk}, d'_{rk}, d'_{bk}, d'_{tk}$  are the distances from the centre of the zone  $k$  to the left, right, bottom and top edges of the base panel respectively.

The equation (6) is used to establish zone similarity between panels

$$S\_Z_j = COMPARISON_{k=1}^{MN}(\{S_i^{new}\}, \{S_k^{base}\}) \quad (6)$$

where  $S\_Z_j$  is the zone  $j$  on the base panel, which is similar to a zone  $i$  on a new panel,  $MN = M \times N$  is the total number of zones on the base panel,  $M$  and  $N$  represent the numbers of zones in row and column within the base panel, and  $COMPARISON$  is the criterion for matching similar zones between a base panel and a new panel.

For laterally loaded masonry wall panels, the Eqns (2) and (3) are proposed as the specific expression of the general Eqns (4) and (5), and the Eqn (7), shown below, is proposed as the specific expression of the general Eqn (6)

$$E_{i,j \rightarrow new}^{k,l \rightarrow base} = \underset{m=1, n=1}{MIN}^{M, N} (|S_{i,j}^{new} - S_{m,n}^{base}| + |S_{i,j-1}^{new} - S_{m,n-1}^{base}| + |S_{i,j+1}^{new} - S_{m,n+1}^{base}| + |S_{i-1,j}^{new} - S_{m-1,n}^{base}| + |S_{i+1,j}^{new} - S_{m+1,n}^{base}|) \quad (7)$$

where  $E_{i,j \rightarrow new}^{k,l \rightarrow base}$  is the minimum error of  $M \times N$  errors in Eqn. (7),  $m$  and  $n$  represent the position of a zone on the base panel,  $i$  and  $j$  represent the position of a zone on the new panel.

Eqn (7) is used to compare the state values of a zone itself and its four neighbourhoods in the new panel with every zone and its four neighbourhood zones on the base panel. An

error value is calculated for each zone by Eqn (7). The zone with the minimum error value on the base panel is defined as the sole similar zone to the zone on the new panel.

## Predicting failure pattern of panel using the above concept and criteria

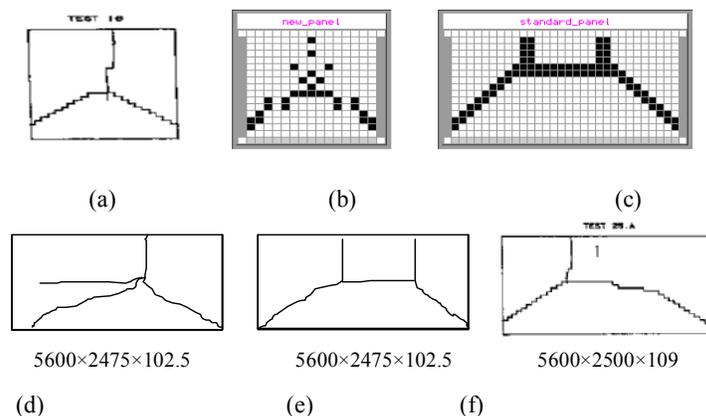
The CA model of the panel, which is described by the Eqns. (2) and (3), and the criteria for matching zone similarity has been used to directly (without using FEA) match the failure pattern of the base panel to a new panel using Eqn (7). This technique includes three steps:

The first step: Establish the CA models of the base panel and the new panel/unseen panel as Fig 3. Eqns (2) and (3) are used to calculate state values of all zones/cells within the base panel and the new panel. Cells in the CA model of the panel are set up to correspond to the mesh division and Gauss points based on the FEA mesh.

The second step: Each zone on the new panel is matched with its similar zone on the base panel using Eqn (7), the zone similarity rule.

The third step: The failure pattern on the CA zone/cell mesh of the base panel is figured based on the failure patterns observed in the lab experiment of the standard experimental panels. Then this cracked pattern is matched on the new panel: if a zone/cell on the base panel is cracked, the corresponding similar zone on the new panel is also assumed to be cracked;

Fig. 4 shows an example that successfully verifies the above procedure.



- (a) The failure pattern of Panel Test 16 recorded in the lab (Lawrence, 1983);
- (b) The failure pattern of Panel Test 16 predicted using the established CA model and based on the failure pattern of the base panel;
- (c) The failure pattern of the base panel summarized from the corresponding experimental panels: (d) Panel SB01 (Chong, 1993), (e) Panel SB05 (Chong, 1993) and (f) Panel Test 29 (Lawrence, 1983).

**FIG. 4.** An Example of Predicting the Failure Pattern of the Panel

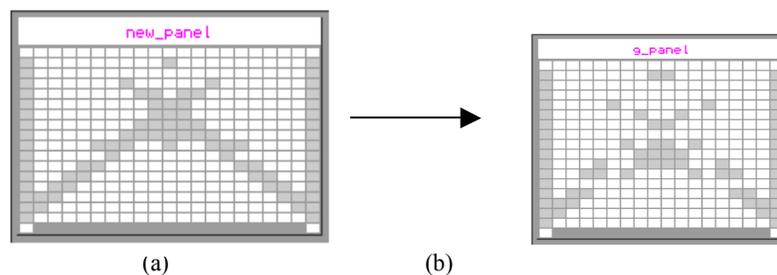
## concept of generalized panel

The above technique has demonstrated that it is possible to directly predict the failure pattern of a laterally loaded masonry panel using the CA model and the existing lab records of a few standard panels. However, the predicted failure pattern does not relate the corresponding failure load which is the most important parameter for the design. The conventional methods are very difficult to deal with such a complex and highly nonlinear relationship between the failure pattern and the failure load of the panel.

Because artificial neural networks (ANN) are suitable for complex nonlinear problems involving a number of parameters and variables, this paper develops a new concept, called **generalized panel**, in order to establish an ANN model to express the relationship between the failure pattern and the corresponding failure load of the panel.

The predicted failure patterns of panels using the CA model can have different mesh sizes. This raises an issue for application of ANN, as the number of cells in input layer for an ANN model, which take their data from the CA model, must be a fixed size.

The concept of generalized panel is to map the failure patterns of panels predicted using the CA technique and expressed with the numerical patterns into a fixed CA mesh which can form the input data for the ANN model. This mapping satisfies the criteria for matching similar zones between the new panels and the generalized panel. In other words, if a zone/cell on the new panel is cracked, the corresponding similar zone on the generalized panel is also assumed to be cracked. In this process the value of a cracked zone/cell is set to 1, and to 0 if uncracked. Since the failure patterns of panels with different dimensions projected on the generalized panel may be the same, this paper multiplies the failure pattern projected on the generalized panel by a dimensionless factor (between 0 and 1) which considers the aspect ratio and the length of the panel, that is, the **dimensionless factor = the aspect ratio  $\times$  the length/the maximum length**. An example is shown in Fig. 5.



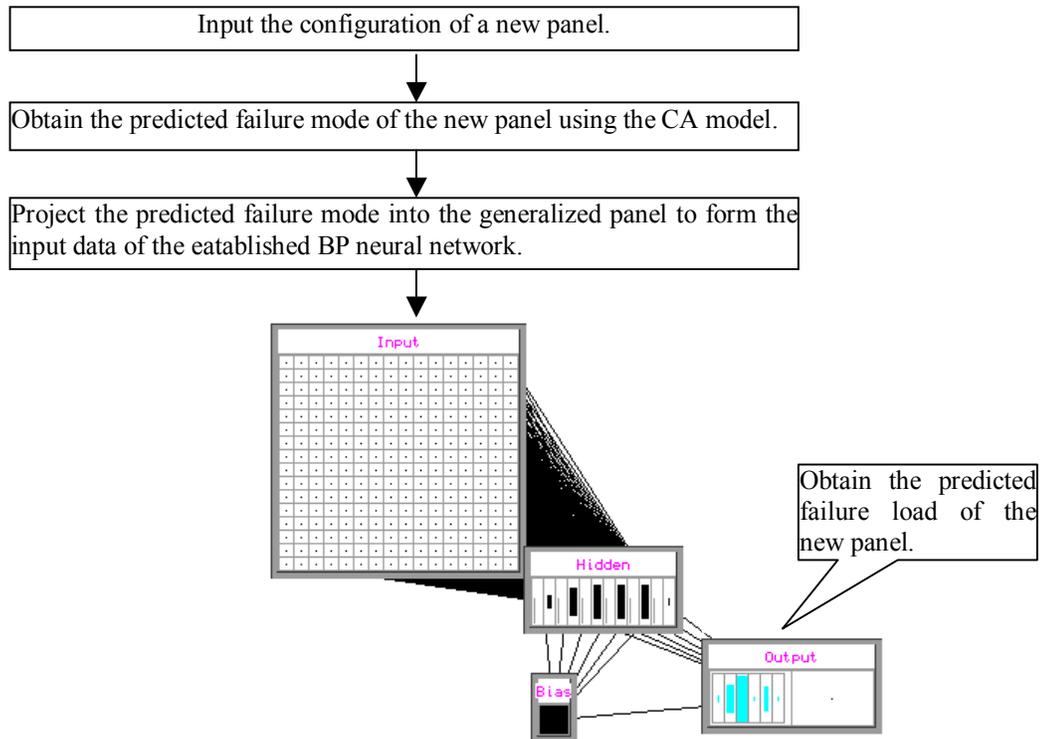
- (a) The failure pattern predicted using the established CA model and based on the failure pattern of the base panel;  
(b) The failure pattern projected on the generalized panel;

**FIG. 5.** An Example of Projecting the Predicted Failure Pattern into the Generalized Panel.

In this research a generalized panel is defined by a panel which is divided to 16 $\times$ 16 mesh. This process is applied to re-mesh all panels to a 16 $\times$ 16 mesh and map the correct failure patterns into this mesh, regardless of these panel sizes.

## The ANN and CA model of predicting failure load and failure pattern of panel

Fig. 6 gives out the ANN model used to predict the failure load of a new panel.



**FIG. 6.** The Structure of the Established BP Neural Networks  
Specification on the BP neural network

The specification on the BP net shown in Fig. 6 is as follows:

Each set in the training and test data for the BP net consists of two parts, input and output. The input is the failure patterns of the panels obtained using the CA model, and the output is the failure loads of the panels. from the lab or the FEA using the stiffness/strength correctors if the corresponding lab record is not available. Failure loads for whole set of panels having various dimensions are obtained using the nonlinear FEA software and correctors. Failure loads for some panels tested in the laboratory by Chong (1993) are also added to this data. The data are used to train the neural net to learn the failure loads of these panels. The train NN is then applied to predict the failure load of the unseen panels.

The choice of the hidden layer of a BP net is usually based on the experience and the comparison of training results to a few different hidden layers with consideration on the learning rate and momentum. This paper deploys 1 hidden layer with 6 cells after verifying 1, 2 and 3 hidden layers with 6, 12 and 18 cells respectively. The learning rate is 0.6 and the momentum is 0.9.. The corresponding training error is 0.035 and the testing error is 0.068

using the training and test data established in this research. The other training and testing errors are more than 0.0367 and 0.1077. Fig. 7 shows the training result of the established BP neural network (Fig. 6).

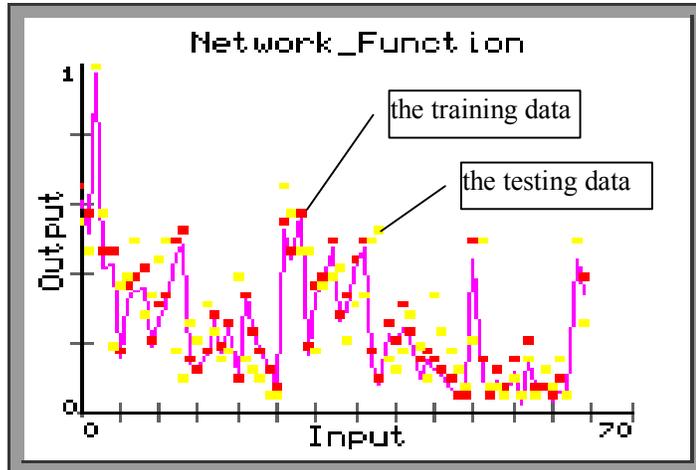


FIG. 7. The training result of the established BP neural network  
 The results predicted using the BP neural network

The Table 1 shows a group of results predicted using the trained BP neural net proposed in Fig. 6.

Table 1. The predicted result of the lab panels

The sizes of the panels tested in the lab (all panels are supported except for their free top edges)	The lab failure loads (kN/m <sup>2</sup> )	The predicted failure loads using the proposed BP net (kN/m <sup>2</sup> )	Errors
2500×2500	7.8	7.736	0.82 %
3750×2500	3.4	3.48	2.35 %
5000×2500	2.7	2.796	3.56 %
5600×2475	2.8	3.024	8%
6000×2500	2.3	2.416	5.04 %

It can be seen: Although the BP net gives outresults in the non- a less conservative prediction, the errors are in the range of allowance; this means the AI technique can replace the conventional techniques to directly predict the failure loads and failure patterns of panels. The table does not list the FEA failure loads for comparison as the author did not obtain the details of the panels enough for the FEA. However, when compared with the existing FEA

results of laterally loaded masonry panels (Chong, 1993), it can confidently comment that the BP net can predict the failure load of the panel more accurately than does the FEA technique.

## Conclusions

1. The new concept of *generalized panel* provides a functional model which can effectively transform a number of numerical patterns with different configurations into a unified numerical format which can be used as the input data of an analytical technique, particularly the AI techniques such as ANN.
2. The proposed ANN model can replace the conventional techniques such as the FEA to predict the failure load of the panel more accurately.
3. The technique developed in this paper can be used as an artificial experimental environment that can replace some of the physical tests for masonry panels, which could be very expensive.

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