Application of the Bid Amount Model to Cost Estimation Systems for Public Works

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ABSTRACT

When government and local agencies order public works, contract officials are mandated to set a threshold price, which is the upper limit of the bidding price for each project, in Japan. We have developed and maintained "a cost estimation system" which computes the threshold price. In this paper, we present a model analysis using the game theory on the basis of micro-economy. The model to examine is based on an auction theory which was developed considering game theory as a basis. We employed "first-price sealed-bid auction". It turned out that it is effective to support bidding and contract procedures complete smoothly, to add the data and logic on which the production cost of a tender company.

INTRODUCTION

An amount of money for the construction bidding and contracting of public works must be less than a threshold price that was set by contract office, according to "Accounting Law" which was established in 1889, in Japan. Although the contract method has continued to improve, an upper limit restricted by the threshold price has not changed still. In order to compute the threshold price, a large quantity of database and a complicated system called "a cost estimation system" which implemented detail logic for calculation are required.

Japan Construction Information Center (JACIC) is a foundation authorized by Ministry of Land, Infrastructure, Transport and Tourism (MLIT) which is responsible for public works administration. In order to promote the smooth procurement of public works, JACIC has been providing such data and system to government and local agencies.

In this paper, to confirm the measures of improvement of "a cost estimation system", we review the "general and administrative expenses" which is an element of the threshold price, from the economic side, by using the game theory based on the micro-economics. In addition, the contents of this paper which we mention are private research results of us, and not orientations of JACIC.

ESTIMATION STANDARD OF PUBLIC WORKS

Figure 1 shows a constitution of "Contract cost", the cause of the threshold price. "Contract cost" consists of "Construction price" and "Consumption tax", and "Construction price" is the sum of "Construction prime cost" and "General and Administrative expenses".

In addition, "Construction prime cost" is the total of "Direct cost of construction" and "Indirect cost of construction." "Indirect cost of construction" is the sum of "Common temporary construction expenses" and "Field management expenses".

Among them, "Construction prime cost" should be determined by the cost of construction of considering, the difficulty of performance, the scale of construction, construction period. But "General and Administrative expenses" greatly depend on a principle of the market, because it is a part of the profit of a bidding company.

In other words, we should examine "General and Administrative expenses" by the economic and management side unlike other elements. Because, "General and Administrative expenses" is decided by the situation of the market or the strategy of a tender company. And other elements are the necessary expenses to carry out the construction.   (Ministry of Land, Infrastructure, Transport and Tourism 2013)
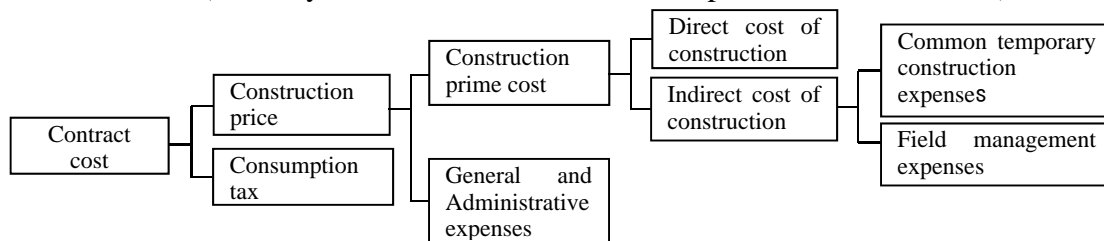

Figure 1. The constitution of "Contract cost"

THEORY OF THE MODEL

In this study, the method of the model that we built is applied from the auction theory based on the game theory.

Game theory (Robert Gibbons 1992). The game theory has been developed based on "Theory of Games and Economic Behavior", established by mathematician John von Neumann and economist Osker Morgenstern in 1944.

The game theory has evolved various fields by the concept of Nash equilibrium which John F. Nash invented. The economic model for "General and Administrative expenses" that we show in this paper is a type of the equilibrium solution called Bayesian equilibrium point.

Auction theory (Miura Isao 2003). In general, the theory which describes the behavior of bidders is the auction theory. This theory, based on the game theory, is used for the designing of the auction method to prevent an illegal act.

Auctions include a public auction system and a closed auction system.

The public auction system is the system that the bidders decide bid prices looking for other bidders' actions each other, like the auction of pictures.

A closed auction system is the system that the bidders decide the bid prices without knowing other bidders' actions each other.

The purpose of our research is to examine "General and Administrative expenses" for "a cost estimation system" of public works. Therefore so we adopted a closed system.

The most typical type of the closed auction is a sealed-bid auction. In the sealed-bid auction, the bidders submit a sealed document to the contract office to not be known by the other tender companies.

There are two kinds of types of the sealed-bid-auction, as follows.

· First-price Sealed-bid Auction (FSA): A winning bidder is the person who puts the highest price in the sealed auction. In public works, the winning bidder puts the lowest price, opposite to the above auction.

· Second-price Sealed-bid Auction (SSA): A winning bidder is the person who puts the highest price in the sealed auction, and the winning bid price is the second highest price.

The threshold price is the upper limit of the bidding prices, and a successful bidding price of the auction is the highest price. Therefore, we chose First-price Sealed-bid Auction (FSA).

Bidding model. We developed a bidding model based on the First-price Sealed-bid Auction (FSA). (Miura 2003)

· First-price Sealed-bid Trade (FST): When the lowest bidding price is less than a threshold price, the company that bids the lowest price wins. And when the lowest price exceeds a threshold price, the bid ends in failure. This bid method is a general competitive bid. And we build the model on the following four assumptions.

· Assumption 1: The contract office and the tender person are risk-neutral.
  This means that a winner of the competitive bid is decided only by a bidding price, not other factors.

· Assumption 2: The tender companies expect the bidding price independently.

· Assumption 3: The tender companies are symmetric.
  This means that a construction amount money of each company follows the same distribution function, and the tender companies are considered to be homogeneous by all other companies. In other word, an estimate cost of successful bid company i (c_i), is a private information of company i, and any other company shall not know exactly. And, we presupposed that c_i follows the probability distribution $F(c_i)$ which the second degree continuation differentiation is possible within a closed-interval [$\underline{c}$, $\bar{c}$], and the tender company expect c_i independently of other costs of the competitors. This is shared knowledge same for all tender companies.

· Assumption 4: A payment from a contract party to the winning company depends on only bidding price. When the contract party selects a winning bid company, he shall consider only a successful bid, and shall not consider a company's

performance, time for delivery, and quality.

If we set b_i to a successful bid price of the winning company, the profit of that company Π_i is as follows.

$$\Pi_i = b_i - c_i \quad \dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots \text{(1)}$$

By the assumptions above, a pricing strategy of the bidder is derived from the Symmetric Nash Equilibrium Strategy.

The following two conditions are needed to build a model which is derived from the Symmetric Nash Equilibrium Strategy, in this case.

- Condition 1: All bid companies which estimate an equal cost (c_i) bid the same bidding prices.
- Condition 2: If the threshold price is set to r, when all the tender companies, other than the company i, set the bidding price $B(c_j; r)$. And $B(c_i; r)$ is the optimal bidding price for the company i.

$B(c_i)/dc_i > 0$, because of the assumption of a bidding price becomes high as the estimate cost is high.

In order to estimate the winning probability of the company i, we set the bidding price of the tender company j, other than i, as follows.

$$B_j = B(c_j : r)$$

If the number of the tender companies set n, a successful bid probability of i is as follows, because it is the possibility of the lowest price in the tender companies. (Gibbons 1992)

$$\text{Prob}\{b_i = \min\{B_1, B_2, \cdots, B_{i-1}, b_i, B_{i+1}, \cdots, B_n\}\}$$

Another expression is as follows.

$$\text{Prob}\{b_i \leqq B_1, b_i \leqq B_2, \cdots, b_i \leqq B_{i-1}, bi \leqq B_{i+1}, \cdots, b_i \leqq B_n\}$$

Prob : the probability of winning a competition.

Furthermore, as $B(c_i)$ is monotonically increasing function of a narrow sense ($B(c_i)/dc_i>0$), if the inverse function of $B(c_i)$ is set $B^{-1}(c_i)$, the upper expression is as follows.

$$\text{Prob}\{B^{-1}(b_1) \leqq c_1, B^{-1}(b_2) \leqq c_2, \cdots, B^{-1}(b_i) \leqq c_{i-1}, B^{-1}(b_i) \leqq c_{i+1}, B^{-1}(b_i) \leqq c_n\}$$

By the assumption 2, the tender companies expect the bidding price independently, and by the assumption3, c follows the probability distribution $F(c)$, the successful bid probability of tender company i is as follows. (Nakabayashi 2012)

The successful bid probability of tender company $i = (1 - F(B^{-1}(b_i)))^{n-1} \dots (2)$

The expected profit of the tender company i , E_π is as follows, from (1) and (2).

$$E_\pi = (b_i - c_i)(1 - F(B^{-1}(b_i)))^{n-1}$$

A necessary and sufficient condition for $B(c_i; r)$ to be the symmetric Nash equilibrium strategy by the assumption3 is as follows. (Iwamoto, Hirota 2013)

$$\frac{\partial E\pi i}{\partial bi}\bigg|_{bi=B(ci)} = 0$$

By a variation of and initial condition of $B(r) = r$, the symmetric Nash equilibrium strategy of the successful company i is as follows.

$$B(c_i; r) = c_i + \int_{c_i}^{r} \frac{(1 - F(x))^{n-1}}{(1 - F(c_i))^{n-1}} dx \quad (c_i \leqq r)$$

Then, we defined that a probability density function is a constant, $f(x) = c$.
Therefore, $B(c_i; r)$ can be described as follows.

$B(c_i; r) = c_i(1 - 1/n) + r/n$ ……………………………………………(3)

As we set the "General and Administrative expenses" to α,

we get $r = c_i (1 + \alpha)$.

$c_i = r / \beta, (\beta = 1 + \alpha)$ ………………………………… (4)

From (3), (4) we can calculate α as follows.

$\alpha = n(r - b)/(b \cdot n - r)$

This α is "General and Administrative expenses" by the symmetric Nash equilibrium based on a market mechanism.

MODEL ANALYSIS

We developed the model using data of the public works of Ministry of Land, Infrastructure, Transport and Tourism. Kanto Regional Development Bureau in the 2013 fiscal year.

The number or target construction is 796.

To verify a calculation result of a model, we compared "General and Administrative expenses" between the calculation result of the model and the estimation standard on which the present cost estimation system is based.

"General and Administrative expenses" based on the estimation standard

In the present system, the rates of "General and Administrative expenses" are determined based on an estimation standard as follows.

Table 1. Rate of General and Administrative expenses

Construction prime Cost	5 million yen or less	exceeds 5 million yen and 3 billion yen or less	exceeding 3 billion yen
Rate of General and Administrative expenses	14.38%	Based on the following calculation formula.	7.22%

[Rate calculation formulas]

$Gp = $ -2.57651 $\times$ Log$(Cp) + $ 31.63531(%) ……………………(5)

Gp：Rate of General and Administrative expenses (%)

Cp：Construction prime cost (yen)

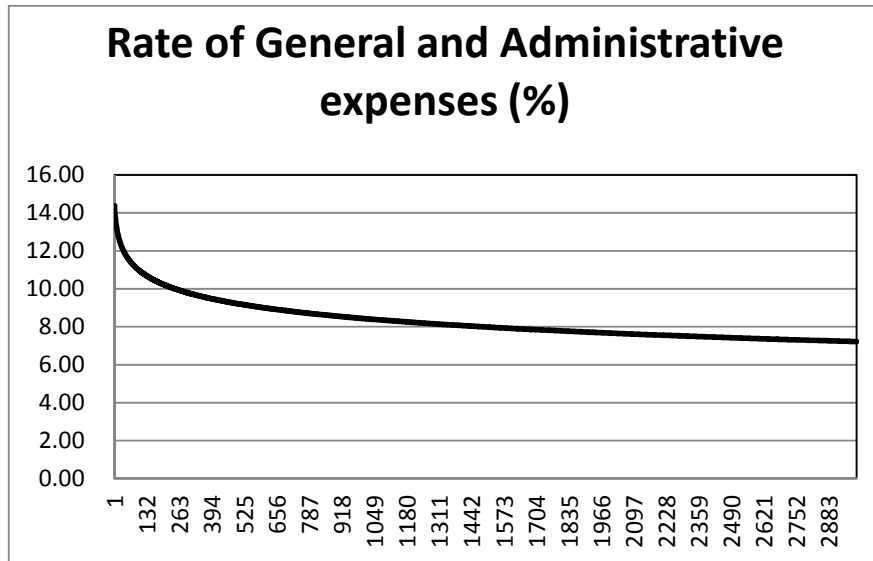Rate of General and Administrative expenses decrease, and converged at 7%

Rate of General and Administrative expenses (%)

Figure 2. Rate of General and Administrative expenses

Result of the estimation form the model. When we calculated a regression shown by (5), we used all the data of above mentioned Ministry of Land, Infrastructure, Transport and Tourism Kanto Regional Development Bureau order in the 2013 fiscal year. These data include the data of the construction prime cost which are 5million yen or less, and exceeding 3 billion yen. So, we re-calculated using the data between 5 million yen and 3 billion yen.

The number of target construction is 781.
The result regression is shown below.
$Gp = -1.08382 \times \mathrm{Log}\,(Cp) + 26.24844$ (%)…………………………(6)
However, Gp: Rates, such as administrative expenses (%)
Cp: Construction prime cost (yen)
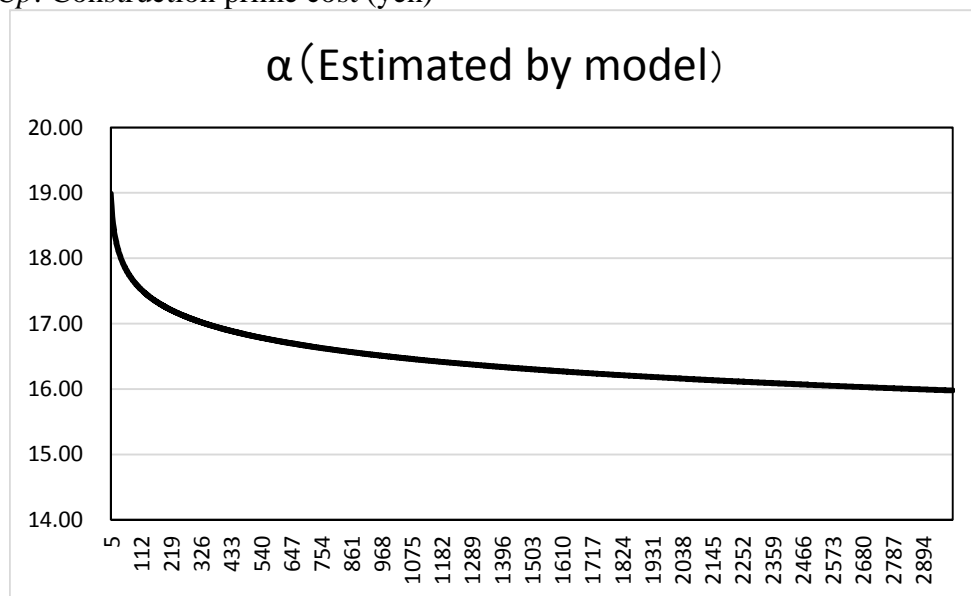
α(Estimated by model)

Figure 3. Result of the model of Rate of General and Administrative expenses

Verification of the model. To verify the model, we compared the values obtained by regression model with the values what the estimation system uses. As we showed in Table 1, "Rate of General and Administrative expenses" is set within 5 million yen and 3 billion yen of the construction prime cost. So, we calculated "the rate of general and administrative expenses" at the 5 million yen and 3 billion yen, by the model.

We show the verification result by Table 2.

The general administrative expense ratio by the model is estimated more highly then the estimation system use, as shown below. The difference between two rates is caused of the correction values according to the conditions of a construction site which is added to present estimation standard. Although correction value can be calculated by investigating an addition result in detail, we cannot obtain the data now.

Furthermore, by the principle of the market, the company may expect profits more greatly than an estimation standard.

These considerations need more informations which needs the check by further special investigation and research, and new knowledge.

Table 2. Compare of General and Administrative expenses

Construction cost	estimation system	model
5 milion yen	14.38(%)	19.08(%)
3 billion yen	7.22(%)	15.87(%)
Difference	7.16(%)	3.21(%)

CONCLUSIONS

It turned out that we can improve the present traditional estimation system to more realistic system based on the situation of the market, by adding the data and logic which reflect the production cost of the tender company.

Moreover, in future Japan, many efficient public-works are needed for maintenance for natural disasters (such as an earthquake), Tokyo Olympic, and the superannuated society's infrastructure, etc.

To these contract matters execute smoothly, it seemed that the estimated price which reflects the market condition quickly, the market factors such as labor unit prices or construction materials unit prices would be needed to be computed. In order to realize these required matters, we think that the economic approach is effective, and this research suggests that such model building is possible.

In this paper, we studied the profit of tender company by building a model. We would like to challenge the further concrete analysis and to arrange the improving point of a bid and contract method of the present public-works.

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