BIM-BASED HYBRID INERTIAL POSITIONING FOR FACILITY OPERATIONS SUPPORT

Saurabh Taneja, Graduate Research Assistant , staneja@andrew.cmu.edu
Burcu Akinci, Professor, bakinci@andrew.cmu.edu
James H. Garrett, Professor and Department Head, garrett@cmu.edu
Lucio Soibelman, Professor, lucio@andrew.cmu.edu
Department of Civil and Environmental Engineering, Carnegie Mellon University, Pennsylvania, USA
Bill East, PhD, bill.W.East@usace.army.mil
Engineer Research and Development Center, Illinois, USA

ABSTRACT

Facility operations often require maintenance personnel to locate critical equipment or find the correct shut-off valve in a mechanical room. Mechanical rooms usually have little to no network coverage. Moreover, facilities usually experience a network outage during building emergencies, such as flooding or power outage. Lack of adequate network coverage limits the deployment of network-based positioning and localization solutions in such environments. Hence, there is a need for an infrastructure-free positioning solution that does not require WLAN network. This paper evaluates algorithms that utilize a navigation network created from BIM-based floor plans for correcting inertial positioning data, which is susceptible to drift errors (Gelb 1974). We have built on top of map-matching approaches that have been used for correcting positioning data from GPS by utilizing road network representations in GIS databases (Scott 1994; Spassov 2007). In particular, we have created a navigation network representation of the indoor environments based on the Medial Axis Transform (Taneja et al. 2011b) for implementing two different map-matching algorithms, namely, a weighted geometric matching algorithm and a topological matching algorithm. We have implemented a prototype that acquires data from a commercial Inertial Sensing Unit, performs map-matching calculations on a portable tablet computer and displays the position output on top of a floor plan on the tablet computer. This paper presents the evaluation of the performance of the two map-matching algorithms in field tests carried out in one of the heavily utilized academic facilities in Pittsburgh, PA. Performances of the algorithms have been characterized in terms of horizontal positioning accuracy of corrected position data (in meters).

Keywords: Facility operations, Inertial positioning, Map-matching, Navigation network.

1. INTRODUCTION

Facility operations often require maintenance personnel to locate critical equipment or find the correct shut-off valve in a mechanical room. Mechanical rooms usually have little to no network coverage. Moreover, facilities usually experience a network outage during building emergencies, such as flooding or power outage. Leite (2009) determined that critical time is wasted on locating shut off valves or vulnerable contents during a building emergency, which causes a considerable increase in the losses, when not done effectively and efficiently, during such emergencies. Lack of adequate network coverage limits the deployment of network-based positioning and localization solutions in such environments. Hence, there is a need for an infrastructure-free positioning solution that does not require deployment of additional sensors. There has been a considerable amount of research devoted to developing novel solutions for indoor positioning for supporting many applications. Liu et al. (2007) carried out an extensive review of the various available indoor localization technologies and the various techniques used for calculating location information from raw data. Taneja et al. (2011a) built on top of the review carried out by Liu et al. (2007) and categorized localization technologies as broadcast-based, motion-based and vision-based technologies. Taneja et al. (2011a) also identified requirements for indoor positioning technologies for supporting O&M field activities and evaluated three positioning technologies, namely Wireless LAN (WLAN), Radio frequency identification tags (RFID) and Inertial measurement units (IMU), based on the identified requirements (Taneja et al. 2011a). In that previous study, it was determined that RFID-based positioning is not a good solution for supporting O&M field activities due to reduction in positioning accuracy over time; WLAN-based positioning provided the best results with 1.5m accuracy for 95% precision, in areas of good coverage (strong signal strength) and; IMU-based positioning gave a high error of 5.2% of the total path traversed (160m) (Taneja et al. 2011a), but it is possible to reduce this error by utilizing the knowledge of the indoor environment geometry and topology (Glanzer et al. 2009).

Although WLAN-based positioning provided good accuracy and precision for supporting many O&M field activities, the coverage of WLAN in areas, such as mechanical rooms, machine rooms and warehouses, can be limited. Little to no network coverage in these areas can impede the accuracy and precision of WLAN-based positioning. Hence, there is a need to develop an indoor positioning approach that is not limited by the deployment of sensor infrastructure, such as WLAN or RFID tags, for achieving required positioning accuracies. Figure 1 presents an overview of the existing indoor positioning technologies based on whether they require sensor deployment or not.

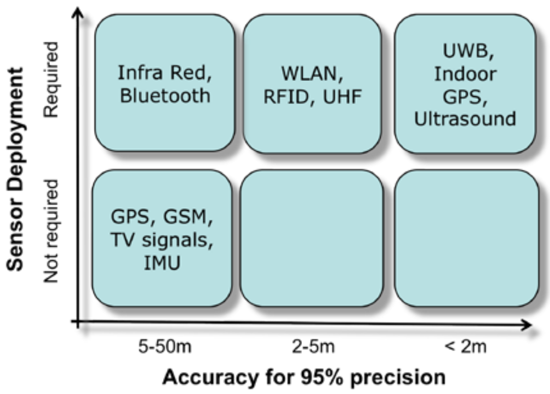

Figure 1. Landscape for indoor positioning technologies

It is clear from Figure 1 that positioning technologies, such as GPS, GSM and IMU, which do not require sensor deployment at the site where users and objects are to be positioned, do not provide good accuracies. Among these positioning technologies, GPS, GSM and TV signal-based positioning are dependent upon the strength of the received signal and poor signal strength in machine rooms and warehouses can further reduce positioning accuracies. IMU-based positioning does not suffer from such external factors, but is prone to internal sensor drift as shown by Glanzer et al. (2009). Many previous researchers have shown that fusing inertial positioning data with information from other sources can improve the IMU-based positioning accuracy (Evennou and Marx, 2006; Wang et al. 2007; Spassov 2007; Glanzer et al. 2009; Walder and Bernoulli 2010). Most of the previous research work that has fused building geometry and topological information with inertial positioning data have only utilized manually created implicit spatial information apart from Glanzer et al. (2009) and Walder and Bernoulli (2010), who have utilized CAD files for correcting inertial positioning data. Hence, we have selected two algorithms that fuse geometry and topology information represented as a navigation network with inertial positioning data to improve IMU-based positioning accuracy. Instead of using only geometric information represented in CAD files, we have decided to use a navigation network representation that resembles a road network representation in GIS databases (Taneja et al., 2011c). The navigation network representation is generated from building information models (BIM) (Taneja et al., 2011b).

In this paper, we have selected and evaluated two fusion algorithms, namely weighted topological matching and topological curve-to-curve matching algorithms. The evaluations were carried out using a test-bed located on the first level of a heavily utilized academic building in Pittsburgh, PA and involved determining the horizontal accuracy of positioning (m). The test-bed formed a loop that included narrow and broad hallways, small passage rooms and a big lounge area. We have created a prototype that implements the selected algorithms and we have evaluated the prototype in our testbed.

2. BACKGROUND LITERATURE REVIEW ON ALGORITHMS FOR FUSING INERTIAL POSITIONING DATA WITH NAVIGATION NETWORK DATA

The problem of fusing inertial positioning data with navigation networks for correcting positioning errors is similar to the problem of fusing GPS data with road network information for correcting GPS positioning errors. Various researchers have proposed a number of algorithms for fusing GPS positioning data with road network information. The broad categories of these algorithms are called map-matching algorithms because these algorithms involve matching positioning data with map information represented in the form of a road network. Quddus et al. (2007) reviewed map-matching algorithms for transportation applications and categorized different algorithms as geometric analysis based, topological analysis based, probabilistic algorithms and advanced algorithms. Geometric algorithms involve matching the position output from a GPS receiver with the nodes or edges in a road network. Mostly geometric algorithms constitute point-to-point matching (White et al., 2000; Spassov, 2007), point-to-curve matching (White et al., 2000; Spassov, 2007) and curve-to-curve matching (White et al., 2000; Taylor et al. 2001; Spassov, 2007). Point-to-point matching involves matching the current position point to the nearest node point in the road network. Point-to-curve matching involves matching the current positioning point to the nearest edge in the road network. Curve-to-curve matching involves matching the shape of the curve of the positioning data from GPS to shape of segments of a road network. Most of the geometric algorithms involve calculating geometric properties such as distance between various entities. Accuracy of geometric algorithms is highly dependent upon the road network shape as well as on the error characteristics of the GPS. Topological algorithms extend geometric algorithms by utilizing the topological connections in the road network data. Topological algorithms check for connections between all possible links and the current link and upon identification of the connected links, geometric calculations, such as point-to-curve matching, are performed (White et al. 2000; Spassov 2007). The performance of topological algorithms can be limited if a wrong link is selected and pursued for some time because of the fact that there might not be any connection between the wrong link and the correct link when the algorithm realizes that current link is not the correct link (Quddus et al., 2003).

Probabilistic algorithms maintain probabilities of the GPS positioning data being present on all the links or edges in a road network. This can be achieved by utilizing the horizontal dilution of precision (HDOP) information from a GPS receiver (Ochieng et al. 2004) or by modeling the errors in the positioning data and utilizing a filtering approach on the uncertainties associated with the positioning data (Woodman and Harle, 2009). Probabilistic algorithms can be accurate, but it is difficult to model the uncertainties in positioning data and when these uncertainties are readily available, such as the HDOP of a GPS, then implementation of a filtering algorithm can cause a lot of computation overhead and consume large amount of time for map-matching. Advanced map-matching algorithms include algorithms based on Dempster-Shafer theory (Yang et al., 2003), Bayesian belief theory (Pyo et al., 2001; El Najjar and Bonnifait, 2005) and fuzzy logic (Quddus et al., 2006; Obradovic et al., 2006). Most of these advanced map-matching algorithms create a rule-base using fuzzy logic or Bayesian beliefs and utilize the rule-base for map matching. These algorithms have the main limitation of creating the rules using GPS positioning data and road network data. Hence, the created rules are dependent upon the error characteristics of the GPS positioning data as well as the characteristics of the road network data.

Due to the fact that matching inertial positioning data to indoor navigation network is similar to matching GPS positioning data, we can utilize the map-matching algorithms proposed in transportation applications domain. At the same time, the differences in positioning data obtained from GPS and an inertial sensor limit the applicability of exactly the same algorithms proposed in the transportation applications domain. For instance, a GPS unit outputs an absolute position estimate in a global reference system with an estimate of the error (HDOP), whereas the inertial sensor only outputs the distance travelled since the last sensor reading and the direction of motion. Hence, an inertial sensor gives a relative position estimate in a local reference system. Moreover, a GPS unit also outputs velocity and heading direction with every position estimate, whereas combining the position estimate of the current time point and the previous time point for the inertial sensor can only derive this information. These factors along with the fact that inertial sensors have drift errors (Glanzer et al. 2009) limit the applicability of the same algorithms proposed for the transport applications domain.

3. SELECTED ALGORITHMS FOR FUSING INERTIAL POSITIONING DATA WITH NAVIGATION NETWORK INFORMATION

Quddus et al. (2007) compared accuracy of various map-matching algorithms for correcting GPS data using road network representations. Based on their review, fuzzy logic and probabilistic map-matching algorithms perform better than geometric and topology-based map-matching ones. The basis for both fuzzy logic and probabilistic algorithms is the information on error covariance or HDOP obtained from GPS. This is probabilistic information, which is not available for inertial sensors. Moreover, a GPS receiver outputs velocity and heading direction for each time step whereas velocity and heading direction for inertial sensors can be calculated from readings of two consecutive time steps. The accuracy of such derived values in turn depends upon the accuracy of the position estimates for an inertial sensor. Due to unavailability of some of the required inputs and the uncertainty of the errors associated with others, we did not consider fuzzy logic and probabilistic algorithms. Quddus et al. (2007) also identified in their review that topological map-matching algorithms perform better than geometric algorithms; hence we selected a topological map-matching algorithm proposed by White et al. (2000) for correcting GPS data. Spassov (2007) showed that a weighted geometric matching algorithm originally proposed by Greenfeld (2002) has good positioning accuracy for inertial sensing based pedestrian navigation. Hence, we also selected the algorithm developed by Greenfeld (2002) and utilized by Spassov (2007) to compare the performance (horizontal positioning accuracy) of a topological map-matching algorithm with a geometric map-matching algorithm in the same test-bed.

Geometric map-matching algorithm involves determining the current link or the edge in the indoor navigation network on which the user is travelling by examining the links or edges connected to the link on which the user was travelling at the previous time step. Further geometric analysis is performed only on the link from previous time step and the links connected to it based on the connectivity weight (Figure 4). Upon the identification of all the links on which geometric analysis has to be performed, proximity weight and orientation weight are calculated. Proximity weight measures the proximity of the inertial positioning data to a link and is represented by Figure 2. Orientation weight measures the orientation match between the link and the line segment formed by inertial positioning data from previous time step and the current time step. Figure 3 represents orientation weight. The overall weight of an edge or a link is found by adding the weights for connectivity, proximity and orientation as shown in Equation 1. Figure 5 presents the flow chart for the weighted topology correction algorithm. The process of map-matching starts every cycle by obtaining the current displacement from an inertial sensor, the edge occupied at the previous time step and the previously corrected position data. All the edges in the road navigation network or the topology graph that are connected to the previously occupied edge are identified. Proximity, orientation and total weight are calculated for all the identified edges. The edge with the highest weight is selected as the currently occupied edge.

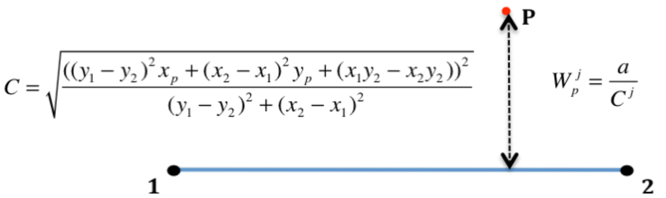

Figure 2. Proximity weight in the weighted topology algorithm

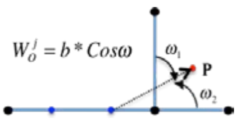

Figure 3. Orientation weight in the weighted topology algorithm

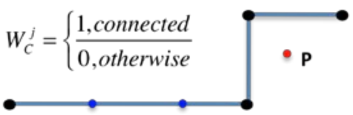

Figure 4. Connectivity weight in the weighted topology algorithm

$$W_T^j = W_C^j + W_P^j + W_O^j$$ Equation 1

The second map-matching algorithm involves matching the curve generated by the raw sensor data with the geometry of the navigation network segments. Figure 6 presents the flow chart for the topological curve-to-curve matching algorithm. The process of map matching starts every cycle by obtaining the occupied edge and the corrected data during the last “t” time steps and the current displacement. The total distance travelled in the last “t” time steps is calculated and all the network sub-sections, starting from the occupied edge from the previous “t” time steps, which are equal in length to the travelled distance, are identified. The curve match weight is calculated as the distance between the curves obtained from the raw sensor data of the last “t” time steps and the network sub-sections, as given by Equation 2. A and B are the two curves in Equation 2 and a(t) and b(t) are coordinate points on respective curves A and B, which have the same parametric value, t. The equation depicts the calculation of the Euclidian distance between the parametric points a(t) and b(t) and summing the distances. The network sub section having the lowest weight is selected as the matched sub-section. The starting edge of the network sub-section is selected as the occupied edge from the prior “t” time steps and the ending edge of the network sub-section is selected as the currently occupied edge. The projection of the most recent inertial positioning data onto the currently occupied edge gives the latest corrected positioning data.

$$\| A - B \| = \int_0^1 \| a(t) - b(t) \| \, dt$$ Equation 2

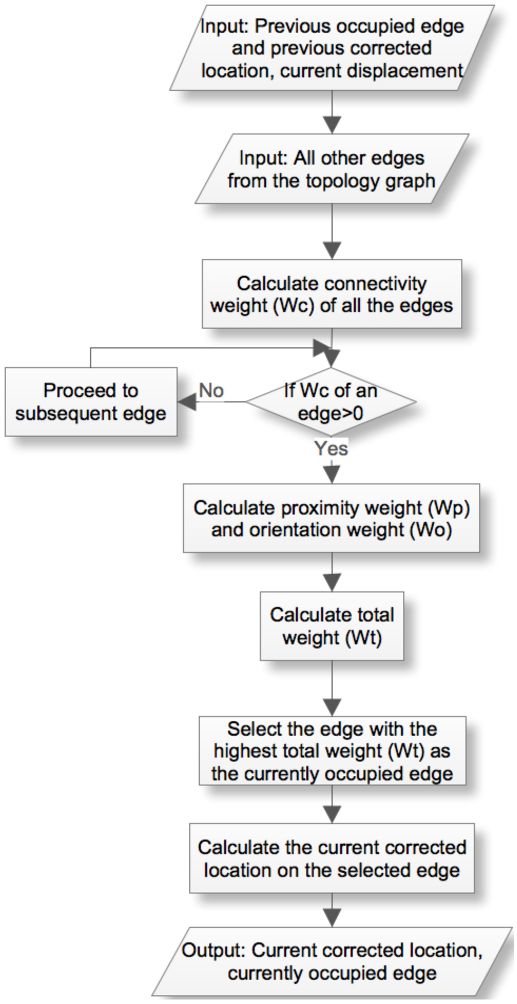

Figure 5. Flow chart for weighted topology map-matching algorithm

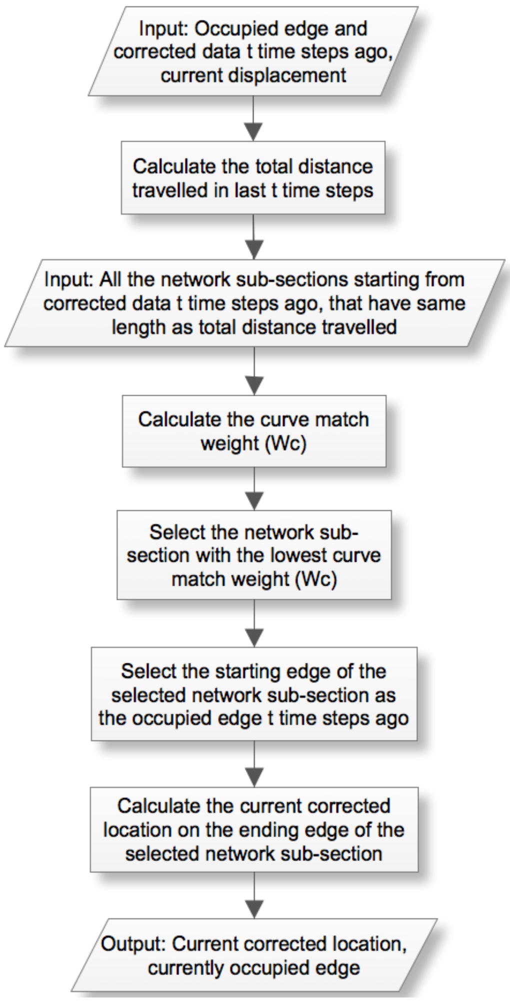

Figure 6. Flow chart for topological curve-to-curve matching algorithm

Both of the selected algorithms have been implemented in the proof-of-concept prototype for hybrid inertial positioning. The next section describes the testbed for evaluating the fusion algorithm implemented in the proof-of-concept prototype.

4. TEST-BED DESCRIPTION AND EXPERIMENT DETAILS

As the test-bed for evaluating the selected algorithms, we have selected a navigation route that loops on the first floor of a university building, that includes narrow and broad hallways, two small rooms and a big lounge area that are used as a passage,. One of the small rooms, used as a passage, contains electronic equipment that can cause electromagnetic interference with the inertial sensor. Such an environment is selected to test the robustness of map-matching algorithms in correcting erroneous positioning data in complex environment with no definite paths (as in the big lounge area) and with the presence of electromagnetic interference from other sources. The test-bed is 100m long and the ground truth data has been collected on 65 points that are spaced 1.5m apart, which is roughly the distance of two human strides (Ladetto et al. 2000). 1.5m distance provides a grid-size to achieve sub-room level accuracies. Figure 7 depicts the test-bed used for evaluating the map-matching algorithms. The dots in the figure depict the starting points in the loop. Two different starting points in the loop were used and multiple sets of data were collected for both the starting points.

Inertial positioning data was collected in the test-bed 30 times by installing the developed prototype on a state-of-the-art mobile computer. Data was collected on three different days during different times of the day. Half of the total data was collected in the test-bed loop by walking in the clockwise direction (15 rounds) and the other half by walking in the anticlockwise direction (15 rounds). This was done to test the effect of any static electromagnetic interference in the environment on the inertial positioning data. The raw inertial positioning data was corrected using the map-matching algorithms developed as part of the prototype. During data collection, the ground truth for the user position was measured every 1.5m in the test-bed by crossing various check points. The map-matching algorithms were evaluated by measuring the horizontal accuracy of the corrected data as compared to the accuracy of the raw sensor data.

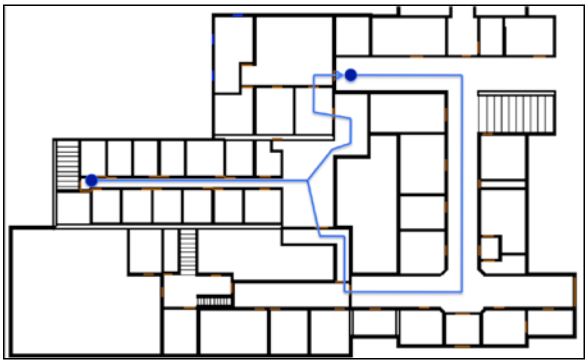
Figure 7. Test-bed for evaluating map-matching algorithms

5. RESULTS

The proof-of-concept prototype implementation involved coding of the two map-matching algorithms described in Section 3, namely the weighted topological algorithm and the topological curve-to-curve matching algorithm. Figures 8 and 9 depict the correction of erroneous inertial positioning data by the weighted topological map-matching algorithm and Figures 10 and 11 depict the correction of erroneous inertial positioning data by the topological curve-to-curve matching algorithm. The red lines in the figures 8-11 are the representation of the indoor navigation network generated based on the Medial Axis Transform (Taneja et al. 2011b). The horizontal accuracy achieved by the weighted topological algorithm, as determined by the evaluations in the test-bed, is low at around a 5m averaged for 30 rounds. This low accuracy is due to high sensitivity of proximity and orientation weights (Figures 2 and 3) to network geometry and sensor characteristics (White et al. 2000). It is clear from Figure 8 that when there is high sensor drift, the weighted topological algorithm is not able to accurately associate the raw positioning data to correct a network edge. Figure 9 depicts that when sensor drift is comparatively low, the weighted topological algorithm performs better. The topological curve-to-curve matching algorithm performs better than the weighted topological algorithm and the horizontal accuracy achieved by topological curve-to-curve matching algorithm is around 3m averaged for 30 rounds. The main reason for better performance of the topological curve-to-curve matching algorithm is the fact that it uses more sensor data (limited recent history) for correcting the raw position estimate at every time step. It is clear from Figure 10 that even during large sensor drift, the algorithm identifies the correct network edge at all time steps except at a couple of instances. Figure 11 depicts that even in the cases of a large sensor drift the algorithm deviates initially, but as it gets more sensor data it is able to identify the correct network edge.

Figure 8. Inertial positioning data correction using weighted topology map-matching algorithm

Figure 9. Inertial positioning data correction using weighted topology map-matching algorithm

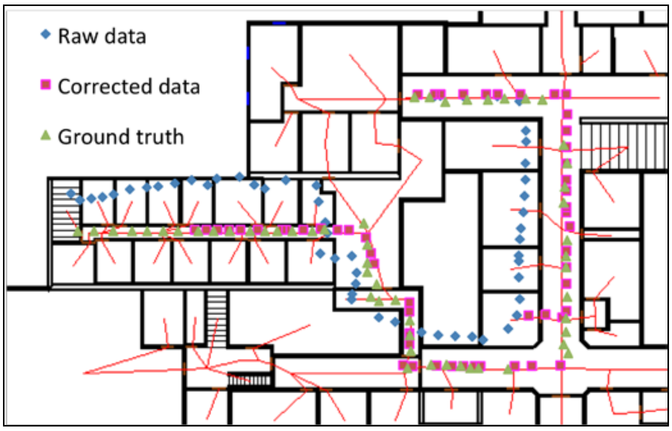

Figure 10. Inertial positioning data correction using topologic curve-to-curve map-matching algorithm

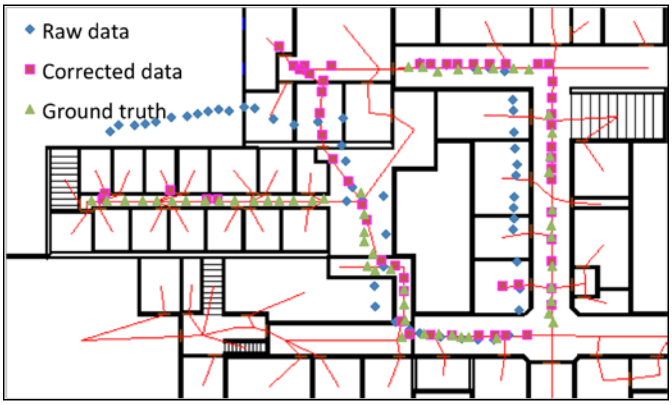

Figure 11. Inertial positioning data correction using topologic curve-to-curve map-matching algorithm

6. CONCLUSIONS

The evaluation of the map-matching algorithms highlighted the fact that there is a high sensor drift in the commercial inertial sensor used in this research. In spite of the high sensor drift, the map-matching algorithms could correct the sensor data utilizing the navigation network data. Geometric matching in the weighted topological algorithm resulted in poor accuracies due to a dense network and high sensor drift, whereas utilizing the shape of the walking path in the topologic curve-to-curve matching algorithm increased the accuracy of positioning correction. There are two interesting observations from the evaluation of map-matching algorithms: 1) high sensor drift should be frequently calibrated using the navigation network or the WLAN-based positioning; and 2) the geometry of the generated navigation network affects the accuracy of the corrected position data. The geometry of the centreline-based navigation network around the corners of hallways results in a longer path than the actual path travelled by a person, which results in poor accuracy of corrected data. High sensor drift induces errors in the calculation of the proximity weight and the orientation weight in the weighted topological algorithm, as well as causes periodic localized deviations in the topological curve-to-curve matching algorithm.

7. LIMITATIONS AND FUTURE WORK

The geometry of the navigation networks around the corners of hallways can cause corrected position data to lag behind the actual position of a user due to smaller actual path travelled by the user as compared to the centreline-based network path. Hence, there is a need to evaluate other types of network representations, such as visibility-based networks, for correcting inertial positioning data. Future work includes development of map-matching algorithms that utilize visibility-based networks and periodic calibration of the drifting inertial sensor position data using WLAN position data.

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