

Single/Dual Variation Approach: A Novel Bridge System Identification Method Based on Static Analysis and Parallel Simulation

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Abstract

Bridge health assessment is an interdisciplinary area of interest for various engineering fields. System identification can be applied to address this assignment, which has been developed for decades. Supported by the modern bridge monitoring system, conventional system identification methods are primarily associated with modal analysis, since the natural frequencies are able to be obtained from the monitored data. Generally, the research focuses are located in simulation results evaluation and monitoring system optimization. The simulation process for system identification is still not prominently improved. This paper proposes a novel simulation based system identification method integrated with parallel processing. Single Variation Approach (SVA) and Dual Variation Approach (DVA) compose this method. In both sub approaches, (quasi-)static simulation is in a parallel processing environment executed instead of dynamic analysis to achieve bridges' responses. In principle, SVA requires lots of monitoring points along the bridge alignment, so that each monitoring section in a bridge can be identified. DVA exploits the enormous traffic load data delivered by Bridge Weigh-In-Motion (BWIM) system. As consequence, the demand of monitoring points in DVA declines significantly, nevertheless, the system identification hereby works still effectively. Five scenario cases are studied to validate the proposed method and to explore its features.

Keywords: system identification, bridge damage, FEM, static analysis, parallel processing

1. Background

The field of system identification uses statistical methods to build mathematical models of dynamical systems from measured data. Since decades, the application of system identification in structure damage detection has demonstrated a wide spectrum. Brownjohn studied ambient vibration of tall buildings to complete system identification by the means of modal analysis (Brownjohn, 2003). Researchers also attempted to compose different algorithms to improve system identification quality. Genetic Algorithm (GA) and Support Vector Machine (SVM) were combined to detect bridge damage (Liu & Jiao, 2011). Convolutional Neural Network (CNN) becoming pretty popular in the last five years was successfully verified to estimate the actual amount of structural damage as well (Abdeljaber et al., 2018; Abdeljaber, Avci, Kiranyaz, Gabbouj, & Inman, 2017). Overall, the mainstream relevant researches focus on combining structural system identification with diverse algorithms or optimizing these algorithms. Meanwhile, dynamical analysis methods including modal analysis are the most widely used method to simulate system assumptions. As the world steps into the epoch of Industry 4.0, several informatics techniques such as cloud computing, virtual reality, internet of things, etc. are already maturely developed for the integration in Architecture Engineering and Construction industry. This paper proposes a novel simulation based system identification method. This method takes advantage of the principle of parallel computing to curtail simulation duration of the entire identification process. Additionally, static analysis supported by a modern structural monitoring system rather than structural

dynamics is executed to identify the local structural damage. The proposed system identification method consists of two sub approaches; both are validated and studied through case study.

2. Bridge Weigh-In-Motion

Bridge Weigh-In-Motion (BWIM) developed by Moses in 1979 measures bridge deformation at ceiling during truck passage and evaluates axle loads beside other characteristics (Petschacher, 2010). The quality requirements have been formulated in project COST 323. In order to get a high accuracy two requirements, which have to be fulfilled, sensitive sensors and appropriate hardware with high sampling rate producing appropriate resolution of signal smaller than $1 \mu s$. Given a truck with three axles passes over a bridge then two times number of axles of unknown has to be solved, i.e. velocity, axle distances and weights. Basis for this type of analysis forms a pre-estimated influence line of the particular bridge. Each analysis of an event starts with estimating velocity, detecting axles, and solving finally a set of equations coming from strain records over time. The estimated axle weights are related to results from a calibration phase at that bridge. A sufficient set of different calibration trucks runs form the basis for accurate results. A BWIM system for production usage involves a lot of quality checks during analysis and is combined with artificial intelligence components to use previous experience in estimating good starting values. BWIM is utilized to collect all load events on different bridge lanes. Road administration is interested in fraction of overloads, extreme loadings, or truck traffic amount, engineers may extrapolate observed events to extreme value statistics and estimate design value for mid-span moment. Measurement combined with sensors of load cycles or bending curvature may complete pictures of the bridge. Figure 1 demonstrates a few sensors and the sensor distribution in a bridge monitoring system.

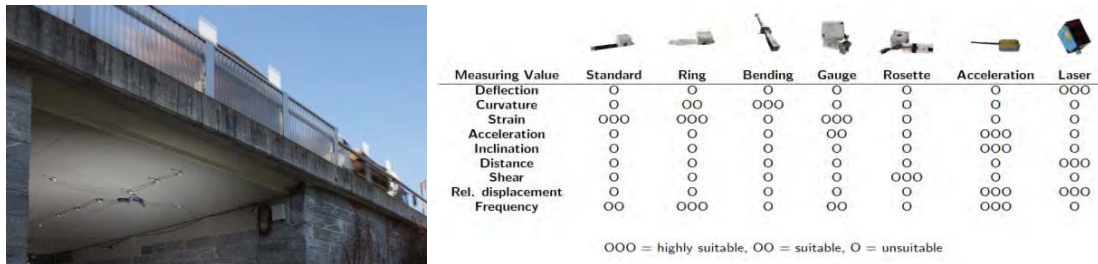


Figure 1: Sensors Installed in a Bridge and Sensors Applied in a Bridge Monitoring System

3. Bridge System Identification

3.1 Stiffness Assumption

The real stiffness reduction β of bridge segments ought to be an irrational number, i.e. there are infinite numbers behind the decimal point. Nevertheless, in civil engineering practice, it is meaningless to detect the stiffness reduction with an extremely high precision, since material laws, structural analysis algorithms and loads all do not reach that accuracy. In order to avoid the over-fitting problem, this system identification approach aims at detecting the current stiffness state α ($= 1 - \beta$) in ten continuous separated intervals with distance 0.1 between 0 and 1 (Table 1). The bottom edge 0.001 in the first interval instead of 0 prevents simulation results from singularity effect due to a complete stiffness loss in bridge segment.

Table 1: Assumption Values and Intervals

Interval 1	Interval 2	Interval 3	Interval 4	Interval 5
[0.001, 0.1]	(0.1, 0.2]	(0.2, 0.3]	(0.3, 0.4]	(0.4, 0.5]
Interval 6	Interval 7	Interval 8	Interval 9	Interval 10
(0.5, 0.6]	(0.6, 0.7]	(0.7, 0.8]	(0.8, 0.9]	(0.9, 1.0]

3.2 Damage Modelling

The repercussions of concrete cracks, which are the major typical damage in bridges, can be reflected through stiffness reduction. Christides and Barr (1983) proposed the one-dimensional theory of cracked Bernoulli-Euler beams to describe this phenomenon. Since then, the mathematical model of concrete crack has been developed intensively for decades (Shah, 1997). Pursuant to the theoretical researches, Firswell and Penny (2002) transformed the mathematical crack model into the numerical computation (Figure 2).

According to the theory of FEM and the modelling philosophies in FEA software, an efficient solution to addressing damage modelling is altering the corresponding parameters in input models of FEA software (Lin et al., 2019). For instance, the reflection of concrete crack could either be a rough factor of the entire stiffness value or weakening some specific material features at the damaged position in the bridge model.

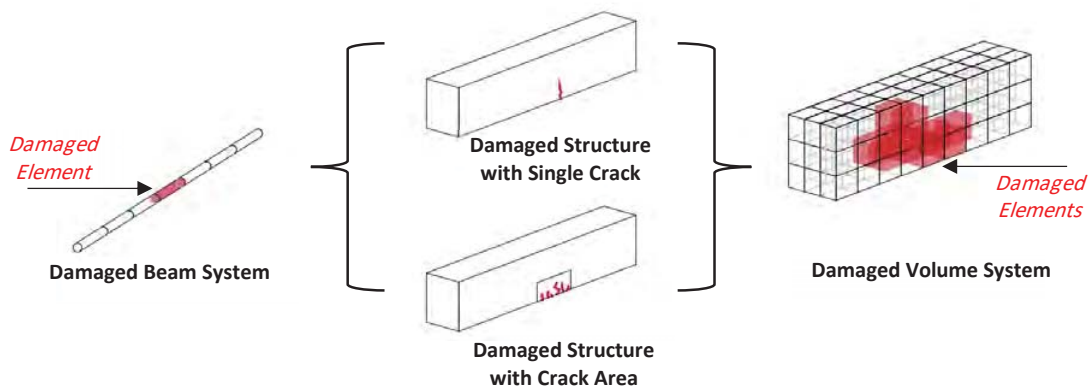


Figure 2: Crack Modelling in FEA Software

3.2.1 System Variants and Parallel Simulation

Figure 3 demonstrates the workflow of the proposed system identification method containing Single Variation Approach (SVA) and Dual Variation Approach (DVA). Both approaches will be described beneath with more details. The concept system variants in SVA means simplify model variants, parameters (e.g. bending stiffness) of which are modified by the initial values from sampling in section 2.1. Besides model variants, system variants in DVA involve load variants as well.

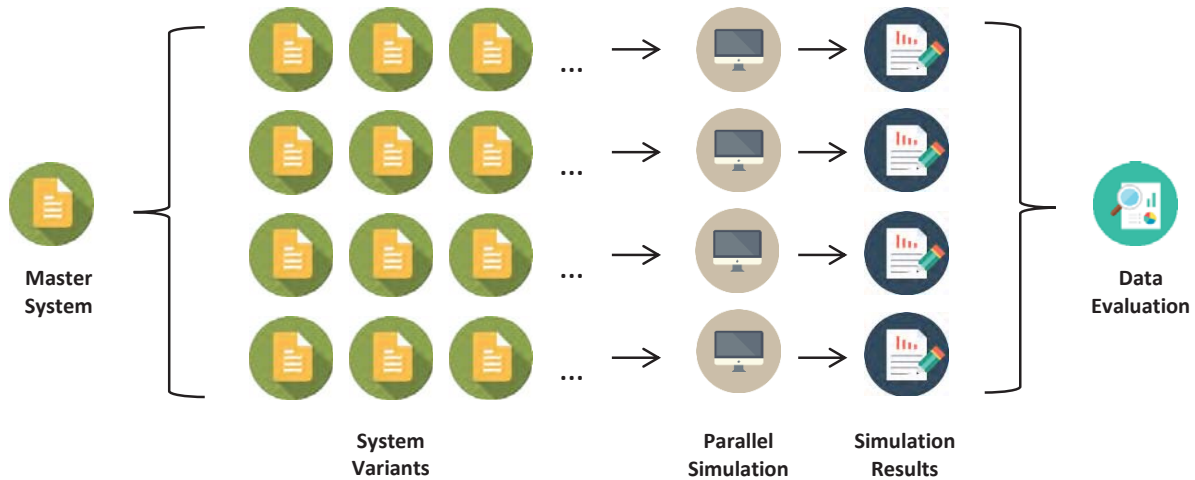


Figure 3: A System Identification Workflow of SVA and DVA

Copious system variants are requested to be generated. The quantity of model variants could reach a giant order of magnitude theoretically, e.g. one million or much larger. To a computer with single processing, the probable overloaded work will cause a lengthy computing duration and even system crash. Therefore, it makes sense to utilize parallel computing technique to handle this problem.

Parallel computing is a type of computation in which many calculations or the executions of processes are carried out simultaneously. As the concept of Big Data arises, parallel simulation or parallel processing appears with an increasing frequency. This technique can either be implemented in one multi-core computer with several parallel coroutines, or be realized through servers in grid or cloud. During the simulation based system identification integrated with parallel processing, system variants are grouped according to the parallel processing quantity, subsequently executed separately and simultaneously, so that the entire simulation duration decreases significantly.

3.2.2 Residual Sum of Squares

A Residual Sum of Squares (RSS) is a statistical technique used to measure the amount of variance in a data set that is unexplained by a regression model. RSS is a measure of the amount of error remaining between the regression function and the data set. A smaller RSS figure represents a regression function that explains a greater amount of the data (Desai, 2011).

RSS is defined by the following function:

$$RSS = \sum_{i=0}^n (\epsilon_i)^2 = \sum_{i=0}^n [y_i - f(x_i)]^2,$$

where n indicates set number of count, y_i and $f(x_i)$ represent the value sets that need to be compared. In statistics, RSS is a basic measure of the discrepancy between the data and an estimation model.

3.2.3 Single/Dual Variation Approach

Through the modern bridge monitoring techniques, engineers are capable to obtain the real-time traffic load data and the corresponding deflection at monitoring points with certain accuracy from captured sensor data. Hence, the Single Variation Approach (SVA) is proposed to address system identification using quasi-static analysis. By means of SVA, system variants are the single argument where variation takes place. The response of damaged bridges due to one load case are then calculated. Afterwards, the simulation results serve for the comparison with deflections from the monitoring data.

The equation of quasi-static analysis is:

$$\mathbf{K} \cdot \mathbf{u} = \mathbf{f};$$

the bridge response vector \mathbf{u} here can be computed:

$$\mathbf{u} = \mathbf{K}^{-1} \cdot \mathbf{f},$$

where \mathbf{K} means the stiffness matrix with reduced/damaged stiffness values, \mathbf{f} is the load vector describing one load case. In Dual Variation Approach (DVA), the equation of quasi-static analysis takes advantage of more load cases:

$$\mathbf{K} \cdot \mathbf{U} = \mathbf{F}.$$

In the equation above, \mathbf{F} is not a vector any more but a matrix consisted of various non-linear related load vectors:

$$\mathbf{F} = [\mathbf{f}_1 \mathbf{f}_2 \dots \mathbf{f}_i \dots \mathbf{f}_n] = \begin{bmatrix} \begin{bmatrix} f_{11} \\ f_{12} \\ \dots \\ f_{1j} \\ \dots \\ f_{1m} \end{bmatrix} & \begin{bmatrix} f_{21} \\ f_{22} \\ \dots \\ f_{2j} \\ \dots \\ f_{2m} \end{bmatrix} & \dots & \begin{bmatrix} f_{i1} \\ f_{i2} \\ \dots \\ f_{ij} \\ \dots \\ f_{im} \end{bmatrix} & \dots & \begin{bmatrix} f_{n1} \\ f_{n2} \\ \dots \\ f_{nj} \\ \dots \\ f_{nm} \end{bmatrix} \end{bmatrix},$$

Because of that, the matrix of deformation \mathbf{U} having the identical size of \mathbf{F} :

$$\mathbf{U} = [\mathbf{u}_1 \mathbf{u}_2 \dots \mathbf{u}_i \dots \mathbf{u}_n] = \begin{bmatrix} \begin{bmatrix} u_{11} \\ u_{12} \\ \dots \\ u_{1j} \\ \dots \\ u_{1m} \end{bmatrix} & \begin{bmatrix} u_{21} \\ u_{22} \\ \dots \\ u_{2j} \\ \dots \\ u_{2m} \end{bmatrix} & \dots & \begin{bmatrix} u_{i1} \\ u_{i2} \\ \dots \\ u_{ij} \\ \dots \\ u_{im} \end{bmatrix} & \dots & \begin{bmatrix} u_{n1} \\ u_{n2} \\ \dots \\ u_{nj} \\ \dots \\ u_{nm} \end{bmatrix} \end{bmatrix},$$

where m is the total quantity of degrees of freedom of the system, n complies with the number of load cases. According to the numbers and categories of data from the monitoring system, the observed bridge responses $\mathbf{u}^r/\mathbf{U}^r$ corresponding to the monitored values $\mathbf{u}^m/\mathbf{U}^m$ are filtered out from the entire simulation results for the purpose of system identification. RSS briefly introduced in section 4.4 is varied in the proposed approach and utilized to assess matching degree of system variants:

$$RSS = \sum_{i=1}^n |d\mathbf{u}_i| = \sum_{i=1}^n [\sqrt{\sum_{j=1}^m (du_{ij})^2}] = \sum_{i=1}^n [\sqrt{\sum_{j=1}^m (u_{ij}^r - u_{ij}^m)^2}].$$

The RSS here is finally calculated as the sum of the error vector norms, so that the impact from positive or negative errors can be prevented. Under the assumption of SVA where n is equal to one, there ought to be however many monitoring data of bridge responses. Therefore, SVA is solely another application of the conventional system identification methods with static calculation and parallel simulation. DVA, in which n in the equations above indicates the number of load cases, is designed to address the scenarios with limited sensor installation places but comprehensive various load cases.

Theoretically, SVA works as a system identification method with strong dependence with monitoring data. In other words, the monitoring system delivering enough data of bridge responses for error calibration in result comparison is a prerequisite for that SVA functions ideally. In the case of limited monitoring data in engineering practice, DVA, an alternative solution, attempts to implement a vast of traffic load from monitoring system in order to identify the best-fit system variants. Several examples are demonstrated beneath for understanding and validation.

3.3 Case Study

3.3.1 General

A master bridge model with constant cross sections is applied to validate SVA and DVA. The freedoms of displacements in three spatial dimensions at both bridge ends are constrained. The isometry, cross section and features of the test bridge are displayed in Figure 4. A 2D FE-system is built by program language Julia based on the bridge material and mechanical parameters. The vertical deflections at each node due to vertical loads are the values of major interest. Considering that, there could be more than one million system variants executed in simulation, 20 parallel processes of Julia established in a local computer are utilized to reduce the entire simulation duration significantly of the designed master bridge in Figure 4.

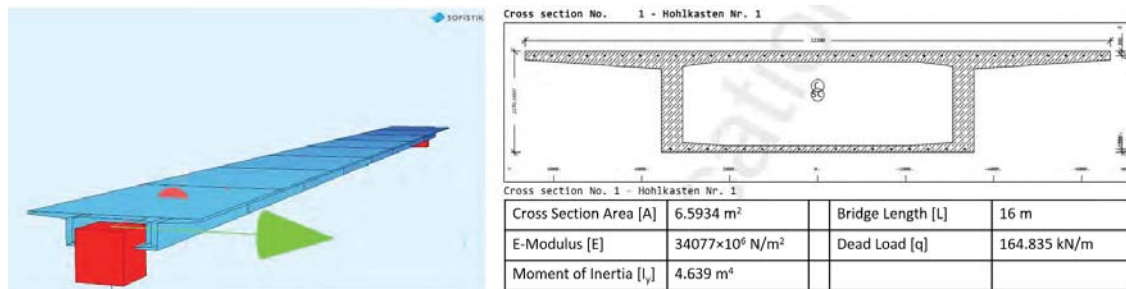


Figure 4: The Bridge Example for Case Study

3.3.2 Single Variation Approach

Figure 5 shows two scenario cases. In Case 1, the bridge system meshed by four beam elements owns three monitoring points and therefore four monitoring sections. Based on the ten sampling groups at each monitoring sections, there are 10,000 system variants in total. The principle of FEM implies that the more finely a system is meshed, the more precise solutions show up after simulation. Hence, the static system in Case 2 is refined by 8 beam elements but still contains 4 monitoring sections in order to observe influence from a more finely meshed monitoring section. As introduced in section 2.5, SVA functions with dependence on monitoring points. One monitoring section either can be an element or consists of several elements for a relatively accurate simulation. The quantity of system variants in SVA corresponds exclusively to the number of monitoring sections. In other words, the bridge system in Case 2 has 10,000 system variants, where each element in one monitoring section has the same stiffness assumption. For both cases in Figure 5, the load vector concerns the combined effect of bridge dead load q and the external single load F .

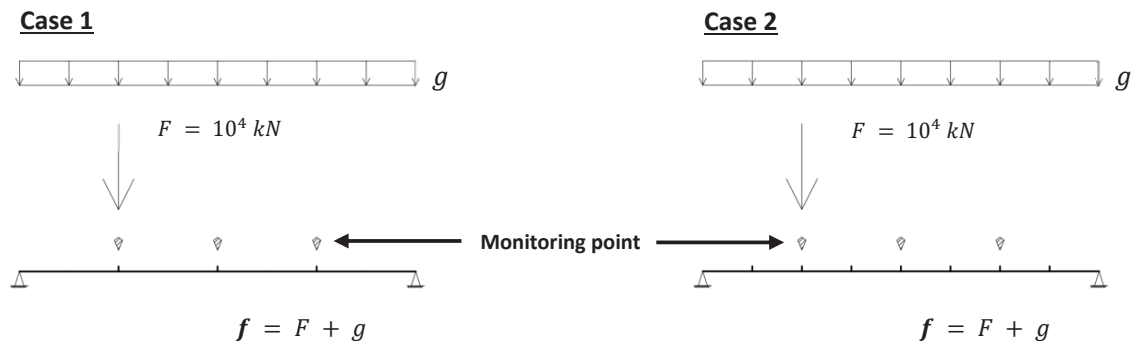


Figure 5: The Bridge Static System for Scenario Case 1 and 2

For theoretical study of system identification, the bridge response based on the assumed stiffness state serves as the assumed monitoring data. Stiffness assumption (S.A. in Table 2) indicates for instance the first monitoring section as well as element 1 in Case 1 has 70% stiffness reduction, so that its current stiffness state amounts to 0.3. The first monitoring section in Case 2 is also assumed to have a 70% stiffness loss, therefore, the belonging elements (Element No. 1 and No. 2) both have 30% of the original stiffness.

Table 2: Simulation Results of SVA for Case 1 and Case 2

E-No.*	Case 1				Case 2							
	1	2	3	4	1	2	3	4	5	6	7	8
S.A.	0.3	0.4	0.2	0.6	0.3	0.3	0.4	0.4	0.2	0.2	0.6	0.6
1 st B.F.S.	0.3	0.4	0.2	0.6	0.3	0.3	0.4	0.4	0.2	0.2	0.6	0.6
S. A.	0.38	0.43	0.22	0.67	0.38	0.38	0.43	0.43	0.22	0.22	0.67	0.67
1 st B.F.S.	0.3	0.7	0.2	0.7	0.4	0.4	0.4	0.4	0.6	0.6	0.5	0.5
2 nd B.F.S.	0.4	0.4	0.2	0.7	0.4	0.4	0.4	0.4	0.5	0.5	0.5	0.5
3 rd B.F.S.	0.4	0.4	0.3	0.6	0.4	0.4	0.4	0.4	0.3	0.3	0.6	0.6

*E-No. : Element Number; S.A. : Stiffness Assumption; B.F.S. : Best-Fit System

If the stiffness assumption has one decimal place, the first best-fit system variant is exact the assumed system. In this way, the system can be identified with no error. If the stiffness is more realistically assumed, e.g. the assumption with two decimal places, the identification results ordered ascendingly by the errors performs complicatedly. The first best-fit system fails to identify all the elements of the bridge system. Thus for the purpose of system identification, the first several best-fit systems are request to be analyzed comprehensively to assess the interval of system stiffness state. Even so, identification works with some error. Considering that the assumption should be irrational numbers, SVA is capable to identify the system with a relative weak accuracy.

3.3.3 Double Variation Approach

Two scenario cases in Figure 6 are studied to validate DVA in system identification, Case 3 has one monitoring point, eight beam elements, four monitoring sections, while three monitoring points are set in Case 4 having the same number of elements and monitoring points. So that 10,000 system variants are executed in parallel simulation. The load matrix F including four different load cases is on the right side of the static equation. The influence of monitoring point quantity on identification quality is hereby observed as well.

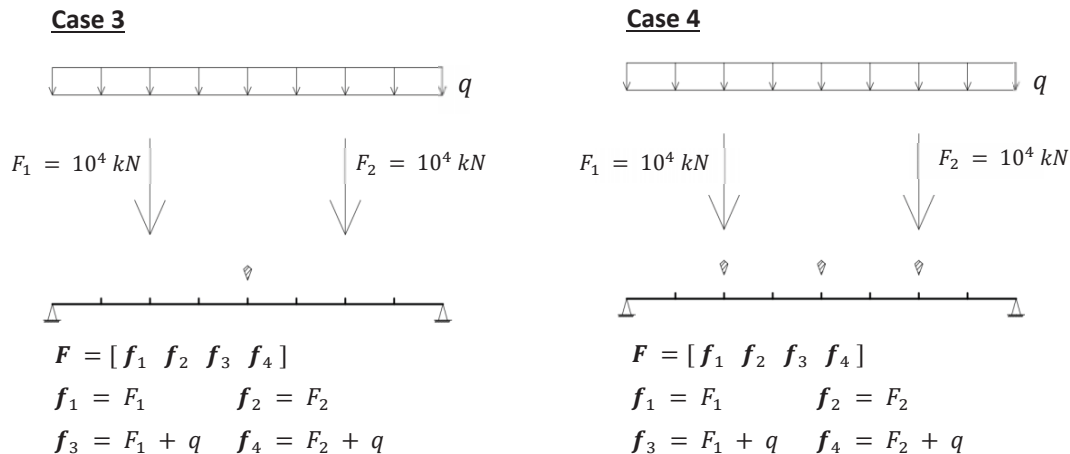


Figure 6: The Bridge Static System for Scenario Case 3 and 4

It is apparent to see that DVA presents a stronger system identification capability than SVA under the stiffness assumption having two decimal places, with which the first best-fit system variant in Table 3 is basically in accordance. Meanwhile the given two scenario cases in Table 3 do not satisfy identifying the system exactly assumed by stiffness with three decimal places. Considering a set of the first best-fit systems, DVA under four load cases offers notwithstanding a rough identification quality. Because the bridge monitoring system is able to deliver surplus traffic load data, an optimization possibility lies in using more traffic load cases for error dwindling. Hence, a static system with 12 load cases (Case 5 in Figure 7) is supposed to be simulated to explore the relationship between load cases and identification accuracy.

Table 3: Simulation Results of DVA for Case 3 and Case 4

E-No.*	Case 3								Case 4							
	1	2	3	4	5	6	7	8	1	2	3	4	5	6	7	8
S.A.	0.38 0.38 0.43 0.43 0.22 0.22 0.67 0.67															
1 st B.F.S.	0.4	0.4	0.4	0.4	0.2	0.2	0.7	0.7	0.4	0.4	0.4	0.4	0.2	0.2	0.7	0.7
2 nd B.F.S.	0.4	0.4	0.4	0.4	0.5	0.5	0.4	0.4	0.4	0.4	0.4	0.4	0.7	0.7	0.2	0.2
3 rd B.F.S.	0.6	0.6	0.3	0.3	0.5	0.5	0.3	0.3	0.5	0.5	0.3	0.3	0.5	0.5	0.4	0.4
S. A.	0.382 0.382 0.435 0.435 0.221 0.221 0.678 0.678															
1 st B.F.S.	0.5	0.5	0.4	0.4	0.5	0.5	0.3	0.3	0.5	0.5	0.4	0.4	0.8	0.8	0.2	0.2
2 nd B.F.S.	0.5	0.5	0.4	0.4	0.2	0.2	0.6	0.6	0.4	0.4	0.5	0.5	0.4	0.4	0.4	0.4
3 rd B.F.S.	0.3	0.3	0.5	0.5	0.2	0.2	0.8	0.8	0.4	0.4	0.4	0.4	0.2	0.2	0.7	0.7
*E-No. : Element Number; S.A. : Stiffness Assumption; B.F.S. : Best-Fit System																

The scenario Case 5 is a bridge static system with two monitoring points installed. The static system is meshed by six beam sections, whereby a monitoring section is equal to a beam element. Thus, there are on million system variants in total since ten values are sampled to estimate each monitoring section. According to the result analysis from cases in Figure 4, 12 load cases including different load locations and load values are of concern in this part of case study.

The first six best-fit system variants under the stiffness assumption with three decimal places are listed in Table 4, where the left part concerns the effect of 10 load cases, the right of 12 load cases. Compared to the corresponding results in Table 3, more load cases is indeed advantageous to system identification. The first two best-fit system variants from Case 5 draw near obviously the assumption, especially if the assumption is situated around the edges of the sampling interval from zero to one. In terms of the number of load cases, the ordered system variants in right table part for 12 load cases do not supply a better identification accuracy than the system variants for ten load cases. It follows that simulation deliberating more load cases brings more effective system identification. At the same time, a collection of redundant load cases cannot lead to an extremely exact identification quality. Hence, an appropriate number of load cases is requested to be determined.

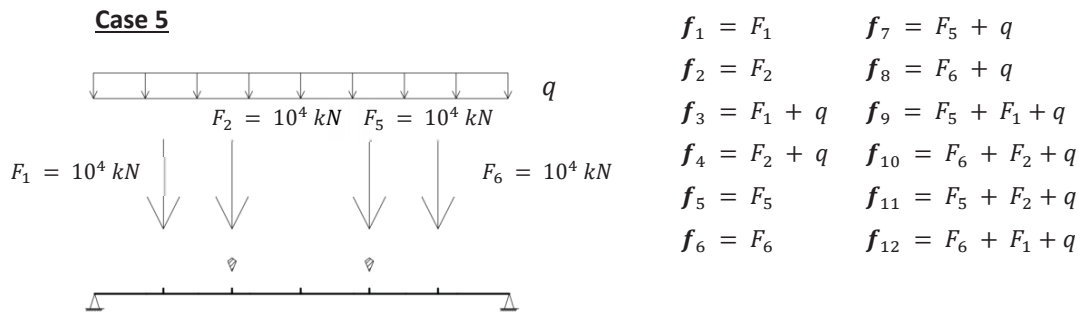


Figure 7: The Bridge Static System for Scenario Case 5

Table 4: Simulation Results of DVA for Case 5

E-No.*	f = f1 + f2 + ... + f9 + f10						f = f1 + f2 + ... + f11 + f12					
	1	2	3	4	5	6	1	2	3	4	5	6
S.A	[0.213 0.425 0.194 0.387 0.642 0.927]											
1 st B.F.S.	0.2	0.6	0.2	0.5	0.5	0.9	0.2	0.6	0.2	0.5	0.5	0.9
2 nd B.F.S.	0.2	0.5	0.2	0.5	0.6	0.9	0.2	0.8	0.2	0.7	0.3	0.9
3 rd B.F.S.	0.2	0.8	0.2	0.7	0.3	0.9	0.2	0.7	0.2	0.6	0.4	0.9
4 th B.F.S.	0.2	0.9	0.2	0.6	0.3	0.9	0.2	0.5	0.2	0.5	0.6	0.9
5 th B.F.S.	0.2	0.7	0.2	0.5	0.4	0.9	0.2	0.7	0.2	0.5	0.5	0.9
6 th B.F.S.	0.2	0.5	0.2	0.4	0.8	0.9	0.2	0.6	0.2	0.5	0.6	0.9

*E-No. : Element Number; S.A. : Stiffness Assumption; B.F.S. : Best-Fit System

3.4 Conclusion and Discussion

As the concept of Big Data arises, massive simulation through parallel processing will show its promising perspective in system identification. The proposed novel system identification method is capable to replace the conventional ones, in which structural dynamic analysis is used to be executed. This paper discusses the feasibility of this method, which has been validated by case study. Both two sub approaches, receptively SVA and DVA, in the proposed method can address the system identification problem. SVA requires a strong data support from a bridge monitoring system. Thus identifying the damage state of each monitoring section works in this case. In addition, finer meshing in monitoring sections does not assist SVA to obtain results that are more reliable. In DVA, the contribution of structural health monitoring to system identification is diminished. Meanwhile, the technique of BWIM is integrated to collect abundant load data of vehicles. By means of DVA, either a monitoring section with elements or a finite element can be regarded as an identification unit. Despite of fewer monitoring points than SVA, DVA brings more plausible system identification results. Evaluating simulation results by RSS performance a rougher accuracy with relative large errors. The chosen best-fit system variants cannot always successfully identify the stiffness statue of all finite elements based on the assumption in Table 1. Therefore, other classification algorithms deserve to be researched to mitigate this shortcoming.

3.5 Implementation in Research Project wiSIB

The acronym wiSIB stands for the German translation of project name “A Simulation and Knowledge Based System Identification Method”. This research project chases two main objectives: 1) establishing a knowledge database describing as many categories of bridge damage as possible; 2) developing a system identification process integrated with parallel computing.

Theoretical argumentation of the proposed system identification methodology from project wiSIB is displayed in this paper. The entire process shown in Figure 4 ought to be realized based on the particular requirements in wiSIB. With the support of FEA software, more kinds of bridge responses can be computed to identify a system as long as the developing bridge monitoring technique offers more direct or indirect useful measurements. Nevertheless, the implementation is conditional. The majority of commercial FEA software must run with legal licenses. While the software is virtually duplicated in parallel processing, either its licenses with unique ID-key are usually uncopyable or these license copies are invalid. To build a parallel processing environment, end users have to pay for a new license for each single coroutine. Although the coroutines of parallel processing can technically increase to a copious number, the cost of software licenses in such circumstance would not be affordable. As a consequences, a grid/could simulation platform based on FEA software functions with confinement of capacity and performance. Therefore, it is meaningful to optimize the Single/Dual Variation Approach to reach the goal of system identification in the orientation of decreasing the operation stress in platform through initiating fewer system variants.

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